



MULTI-SCALE ANALYSIS OF ENGINEERING SURFACES

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ABSTRACT:

Conventional surface characterization techniques involving random process analysis are limited in characterizing multi-scale surface features relevant to manufacturing processes and functions. This paper introduces a novel technique for multi-scale characterization of engineering surfaces by applying wavelet transform. The main advantages of wavelet transform over other existing signal processing techniques are its space-frequency localization and multi-scale view of the components of a signal. Utilizing these properties of wavelet transform, we can effectively apply multi-channel filter banks to the surface data and link the manufacturing and functional aspects of a surface with its multi-scale features. Surfaces produced by typical manufacturing processes are analyzed using wavelet transform, and the usefulness of wavelet transform in the multi-scale analysis of engineering surfaces is demonstrated.

1. INTRODUCTION

It is well known that an engineering surface is composed of a large number of length scales of roughness that are superimposed on each other. These multi-scale roughness features are related to different aspects of the processes the surface has undergone, and influence the functional performance of the workpiece. A comprehensive and efficient characterization of these multiple features therefore provides an important link between manufacture and function.

Conventionally, a surface is viewed as a system of stationary random variables for which various statistical parameters and functions based on random process theory are used for characterization. By means of spectral analysis using Fourier transform, many aspects of manufacture can be identified and used for process diagnostics. For example, periodic surface features can be attributed to vibration of machine tool, tool feed, or the degeneration of the cutting edge. Whereas the relative contribution of white noise may be traced to material fracture caused by built-up edge or the general worsening of the machine condition [1]. In Nayak's random process model [2], the variance of surface height, slope, and curvature can be determined from the zeroth, second, and fourth moments of the surface's power spectrum. Other methods of random process analysis include correlation analysis, time series modelling, etc.

Although it has been widely used in surface metrology, the random process technique is insufficient in addressing the non-stationary and multi-scale nature of rough surfaces. Surface topography inherently contains dynamic information useful for process monitoring and control. However, spectral analysis using Fourier transform only gives time-averaging frequency information without indicating the location of the frequency event. It provides little information about the evolution of the spectrum which usually indicates the gradual degeneration of the process. In the case of an abrupt change in the process, the original dominant spectrum

will be obscured by the broad spectrum excited by the fault signal. It is also found that the statistical properties of a surface are closely related to the scale at which they are assessed. The conventional method is not efficient in separating the effects of the multi-scale features due to the fact that Fourier analysis treats all frequency events with an equal resolution in both space and frequency domains. Nevertheless it is obvious that a sharper resolution in time is needed to resolve fine structures such as the deep scratches on plateau-honed surfaces, while a higher resolution in frequency (which, according to the uncertainty principle, requires a longer sample length) is needed for the larger structures.

What is needed is a space-frequency representation that provides the spectral information on a space-frequency plane which will simultaneously track the multi-scale events over space. To assess the multiple scale structures with reasonable accuracy, the representation also needs to have a flexible space-frequency resolution.

Impressive progress has been made in recent years in exploring the decomposition of signals by space-frequency and space-scale analysis [3,4]. Whitehouse and Zheng [5,6] examined the possibility of using Wigner distribution and ambiguity function in characterizing surfaces, and concluded that the Wigner distribution (WD) offers a likely chance of being useful as a process monitoring tool. However, there is a trade off between good space-frequency resolution and substantial interference terms in using the WD function due to its bilinear structure. In this study, a linear space-scale representation called wavelet transform is chosen as a tool for the multi-scale analysis of engineering surfaces. This novel mathematical and signal processing tool is being used by researchers in acoustics, speech analysis, image processing, pattern recognition, texture analysis, radar processing, medical imaging, fingerprints analysis, seismology, and so on. The body of knowledge on this subject is still expanding rapidly. Its non-stationary and multi-scale view of a signal offers the potential for characterizing multi-scale features of

surfaces without the inadequacies of traditional methods.

The purpose of this work is thus to investigate the potential use of wavelet transform for surface characterization with an emphasis to link the manufacture and function of a surface with its multi-scale structures. An extensive investigation on two- and three-dimensional surface characterization using the wavelet technique was carried out in [7]. In this paper, however, the results are given primarily on two-dimensional surface analysis. The concept of space-frequency representation by wavelet transform as applicable to surface metrology will be briefly reviewed first and then the possibilities of applying multi-scale analysis by wavelet transform to engineering surfaces will be investigated. Functional filtering, process monitoring and diagnostics, and characterization of surface profiles are areas selected for multi-scale analysis in this study.

II. MULTI-SCALE REPRESENTATION USING WAVELET TRANSFORM

As previously stated, a comprehensive analysis of a natural signal must provide us with both the frequencies of the signal and their location. An attempt to use appropriate basis functions to match these two kinds of information results in the introduction and development of the wavelet transform. Figure 1 shows the basis functions of three types of transforms and illustrates the need for the wavelet transform.

The Fourier transform (FT) for a continuous time signal is defined as

$$\hat{x}(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (1)$$

which uses infinitely long complex sinusoids as its basis functions. It is well suited to the study of periodic stationary signals. However, due to the infinite extent of the basis functions, it supplies little information on the location of the frequencies. Any time-local information (e.g., an abrupt change in the signal) is spread out over the whole frequency axis.

To achieve time localization with the spectral information retained, windowed complex sinusoids as shown in Figure 1b were introduced as basis functions by Gabor. This led to a space-frequency representation called short-time Fourier transform (STFT) with both location and frequency as its arguments:

$$T(\omega, \tau) = \int_{-\infty}^{\infty} e^{-j\omega t} w(t - \tau) x(t) dt \quad (2)$$

where $w(\cdot)$ is an appropriate window like a Gaussian function. The short-time Fourier transform provides us with a view of the signal through a time-frequency window with its time-frequency resolution defined by each of the cells shown in Figure 1b. It should be noted that the resolution of the analysis is the same at all locations in the time-frequency plane due to the use of a single window function. The short-time Fourier transform appears to be the best strategy for the analysis of phenomena where frequency events have more or less the same characteristic lifetime, in which case a fixed resolution in time would be sufficient. However, most real signals, as in the case of rough surfaces, do have features of very different scales and a single optimal resolution cannot be defined. Furthermore, the uncertainty principle excludes the possibility of having arbitrarily high resolution in both time and frequency, since it states that the time-bandwidth product $\Delta T \cdot \Delta f$ of possible basis functions must be no lower than $1/4\pi$, where ΔT and Δf are the rms values of the function and its Fourier transform.

The disadvantage of the short-time Fourier transform being its inadequacy for analyzing multi-scale signals due to its fixed resolution in the space and frequency domains, the solution is to have a flexible time-frequency resolution by trading resolution in time for resolution in frequency. That is, the window should narrow (zoom in) at high center frequency (small scale) to give a better accuracy, and widen (zoom out) at low frequency (large scale) to give accurate frequency information. This is exactly what is achieved by wavelet transform (WT), where the basis functions are obtained from a single prototype wavelet (or mother wavelet) $\psi(t)$ by translation and dilation (or compression):

$$\psi_{a,b}(t) = \sqrt{a} \psi(a(t-b)) \quad (3)$$

where a and b are both real numbers quantifying the scaling and translation operations. In this study, we are only interested in discrete orthonormal wavelet transform, where the time-frequency plane is discretized based on a logarithm sampling. By substitution of $a = 2^m$ and $b = n \cdot 2^{-m}$, $m, n \in \mathbb{Z}$, the basis functions become

$$\psi_{m,n}(t) = 2^{m/2} \psi(2^m t - n) \quad (4)$$

Figure 2 shows the effects on the shape and location of a mother wavelet (for example, a wavelet of order 4 given by Daubechies) by the operations implied by equation (4). As a

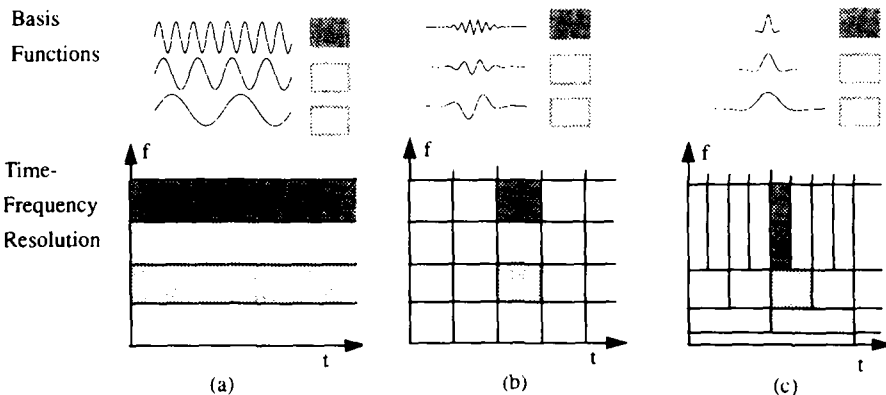


Figure 1 A comparison of the basis functions and time-frequency resolutions of FT, STFT, and WT. (a) Fourier transform. (b) Short-time Fourier transform. (c) Wavelet transform.

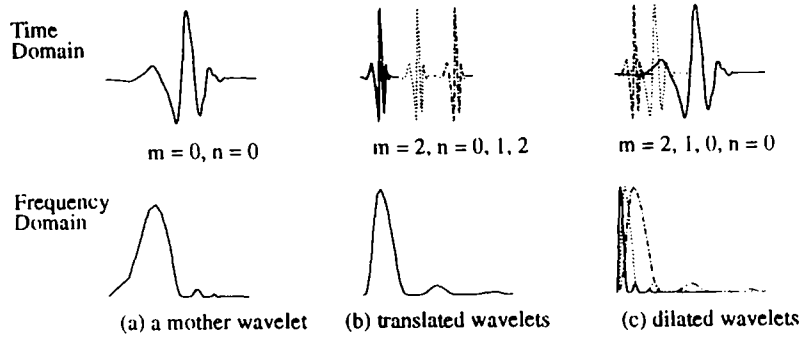


Figure 2 Construction of Dyadic Wavelet Bases (Daubechies' wavelet of order 4).

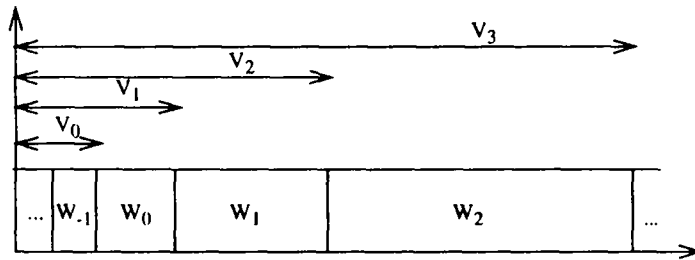


Figure 3 Division and nesting of the space $L^2(R)$ in the frequency domain.

result, the wavelet basis constructed this way forms a flexible time-frequency window localized as one of the cells in Figure 1c on the time-frequency plane. With the basis functions so constructed, the discrete wavelet transform of a signal $x(t)$ is defined as

$$W(m, n) = \langle \bar{\psi}_{m,n}(t), x(t) \rangle \quad (5)$$

where $\bar{\psi}_{m,n}(t)$ is the conjugate of the wavelet functions. In interpretation, the parameter m is referred to as decomposition level correlated to the frequency band assessed, and n indicates the steps of the translation. It is proved by Daubechies [8] that compactly supported mother wavelet $\psi(t)$ can be designed such that the set in equation (4) is orthonormal and complete, which implies a stable and unique reconstruction exists and is given by

$$x(t) = \sum_{m,n} W(m, n) \psi_{m,n}(t) \quad (6)$$

Discrete wavelet transform using orthonormal basis functions is called discrete orthonormal wavelet transform.

The wavelet representation defined by equation (6) allows us to interpret the signal at a certain location t as the weighted sum of the contributions at various dyadic scales around that location. It's distinct in that the wavelet bases $\psi_{m,n}(t)$ have time-widths adapted to their frequency. At the lowest level the wavelet basis covers the whole data, and allows us to examine the stationary feature of the signal present over the whole recorded time period. At each consecutively higher level, the support of the basis function is halved so that we are exposed to smaller and smaller features of the signal. Also note that the translation step is halved as the extent of the basis wavelet is halved. This indicates there is no overlap between the supports of the translated wavelets, hence there's no redundancy in the information covered. It is clear that the property of good time-frequency localization

with a flexible time-frequency resolution of the discrete wavelet transform makes it superior to the other two transforms in resolving features of very different scales.

From a signal processing point of view, a wavelet is a bandpass filter as indicated by the frequency domain representation of the wavelet shown in Figure 2. As shown by Figure 2c, the operation of discrete wavelet transform can thus be interpreted as filtering the signal with a bank of octave bandpass filters, followed by sampling at the respective Nyquist frequencies [9]. Effectively, the discrete wavelet transform divides the spectrum space into subspaces denoted by W_j as illustrated in Figure 3. By adding higher frequency bands, one adds details, or resolution, to the signal. Hence we can also get the outputs of a lowpass filter bank by successive approximation of the signal. The nested subspaces denoted by V_j in Figure 3 are the frequency bands covered by the lowpass wavelet filters.

A pyramidal algorithm is found in [10] for efficiently evaluating discrete wavelet transform. In this study, we have implemented various wavelet representations showing the output of passing the signal through lowpass, highpass, and bandpass wavelet filter banks. Examples are given in the next section where various engineering surfaces are analyzed.

III. MULTI-SCALE ANALYSIS OF ENGINEERING SURFACES USING WAVELET TRANSFORM

Wavelet transform enables multi-channel filtering and the use of this capability is explored for a surface profile measured from an electro-formed surface comparison standards patch, representing a turning process. The data were originally measured at a $0.25 \mu\text{m}$ sampling interval for a length of 17.50 mm so that substantially long pattern as well as fine structures are covered by the sample. To suit the data for wavelet analysis, they were down sampled such that every two adjacent points are $4 \mu\text{m}$ apart and then 4096 consecutive points are chosen for analysis. Any linear trend was removed

by fitting a least square line to the data before any analysis is performed.

An examination on the power spectrum of the measured profile using FFT indicates that there are two dominant harmonics present in the profile, one at 234 μm , and the other at 3.277 mm. It is also shown that the surface contains spectral events of a wide range of scales. However not all these structures except for the two major periodical components are distinctively represented. The majority of minor irregularities are averaged and only serve as a background to make the few major harmonics stand out. In process diagnostics, the occurrence of small structures need to be brought to concern. For example, the scars of the tool tip, and the built-up edge on the surface seriously affect the tool's service life. It is therefore desirable for a space-frequency analysis to be able to resolve the whole range of structures with reasonable distinction. One practice in surface analysis is to pass the data through a filter, for example, an ISO

gaussian filter [11], with a presumed standard cutoff wavelength. Figure 4a shows the output of such an operation with the cutoff wavelengths as 0.08mm, 0.25mm, 0.8mm, 2.5mm, and 8mm. It seems that a cutoff wavelength of 0.8mm is good for separating the longer wave from the shorter wave and a 0.08mm cutoff separates the periodical components from the irregularities reasonably well. However, in practice, usually only one of the standard cutoffs is chosen with little consideration in selecting the "right" cutoff to match the features relevant to application. Even if a comparison like that in Figure 4a is to be attempted, five repetitive filtering operations need to be carried out sequentially, which requires considerable amount of time.

Multi-scale analysis by wavelet transform, on the other hand, offers much more freedom in selecting the cutoff wavelength and gives higher resolution for the fine irregularities without ignoring the relative large structures.

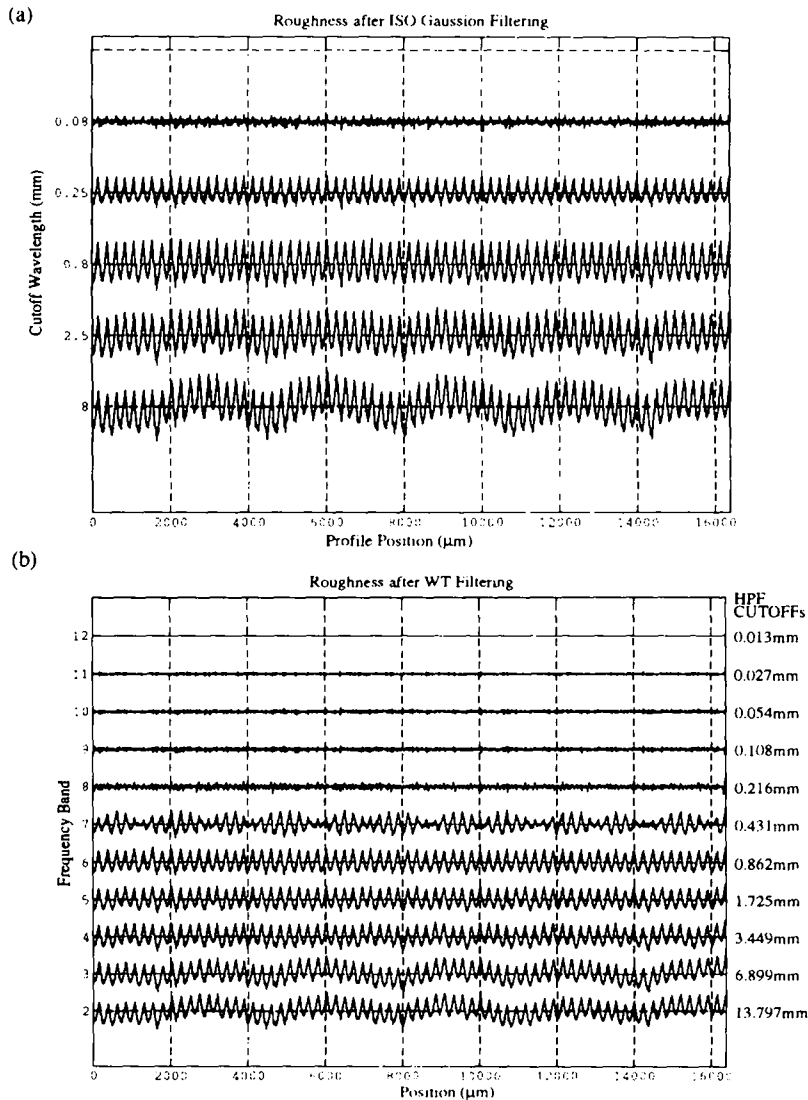


Figure 4 Comparison on the filtering of a surface profile by multiple ISO gaussian filters and multi-channel wavelet filter bank. (a) Roughness after ISO filtering. (b) Roughness after filtering using wavelet transform.

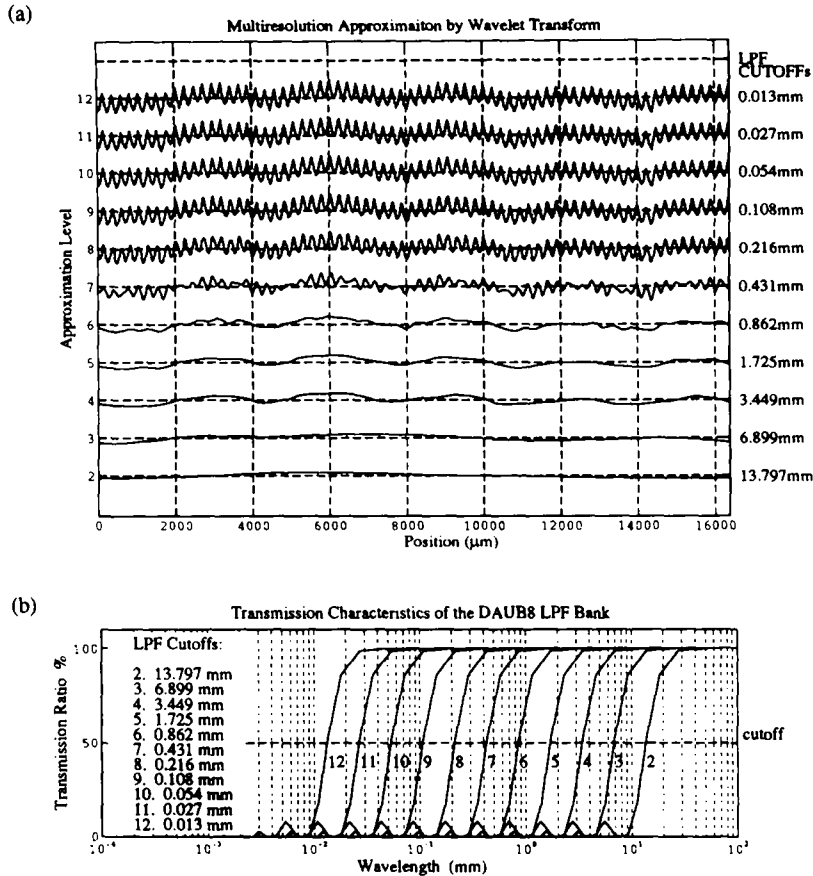


Figure 5 (a) Multi-scale approximations of rough surface by wavelet transform.
(b) Transmission characteristics of the multi-channel lowpass wavelet filter bank used to get the results in (a). The numbers 2, 3, ..., 12 labeled under the curves indicate the respective resolution levels of the filters.

In a single processing, the multi-scale analysis gives the outputs of passing the original profile through the banks of octave lowpass, highpass, and bandpass filters. Figure 5a shows the multi-scale approximations of the profile using wavelet transform, with the closest approximation being the original profile. The x axis represents spatial position, and the y axis denotes increasing resolution level. The frequency band level determined by the parameter m in equation (4) is labeled at the left side of the plot, whereas the lowpass filter cutoff wavelengths at 50 percent transmission points are labeled beside each corresponding level on the right side. The corresponding multi-channel lowpass filter bank transmission characteristics are shown in Figure 5b. The roughness profiles obtained by this filter bank are shown in Figure 4b. The efficiency of multi-channel filtering using wavelet transform compared with conventional surface filtering techniques is clearly illustrated.

By observing the multi-resolution approximations, we can have an overall view of the structures involved in the profile. Specifically, we can see that approximations up to level 6 give the general trend of the profile, which implies the machine condition or tool setup condition in the production of the surface. Up until level 9 of the approximations, the characteristic tool mark formed by clean turning tip is well reconstructed. Above level 9, the irregular structures arising from the degeneration in tool condition are gradually shown. Furthermore, from the wavelet transform representation, we

could divide the frequency bands of the turning process into three divisions based on the causes in manufacture. That is, the initial approximation level and the detail levels up to level 5 constitute the region indicating the machine condition; details in level 6, 7, and 8 together make up the basic tool mark of turning process; and level 9 and above give the irregular components caused by tool wear. The profiles represented by these three divisions of the frequency band are shown in Figure 6. This has some implications in process monitoring and diagnostics. First, the process can be monitored at a certain level depending on the need to identify changes in the manufacturing process. Any change in the amplitudes or spacings of the wavelet details offers dynamic information on the manufacturing process. Second, any change in the process can be traced back to one of the frequency band and the possible cause can be located and analyzed. It is also possible for such a multi-scale analysis to provide the profile of suitable scales for predicting the functional behavior of the surface in such applications as contact, vibration, adhesion, etc., where the sizes involved in the interaction between surfaces make difference.

In real-time process monitoring, the time-frequency localization property of wavelet transform can be utilized for capturing any change in the process on the fly. The idea is illustrated in Figure 7, where a degenerating turning process is simulated and the capability of wavelet transform in detecting the degeneracy is shown. It is clear that detail coefficients at

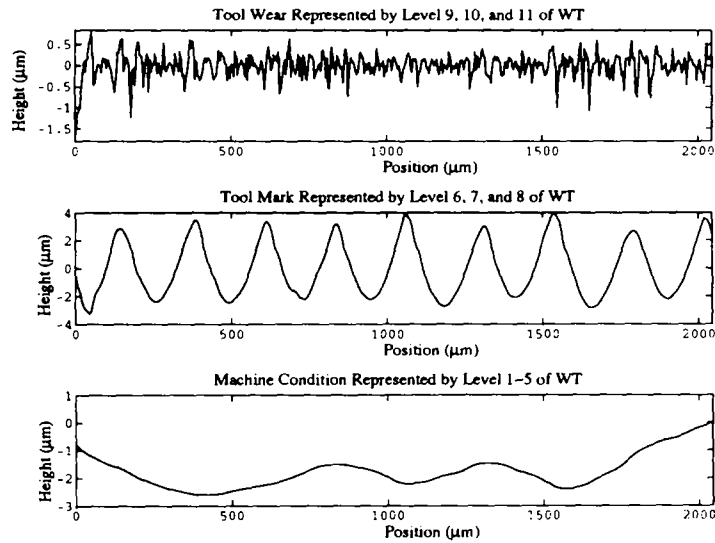


Figure 6 Separating the surface profile based on the causes in manufacture.

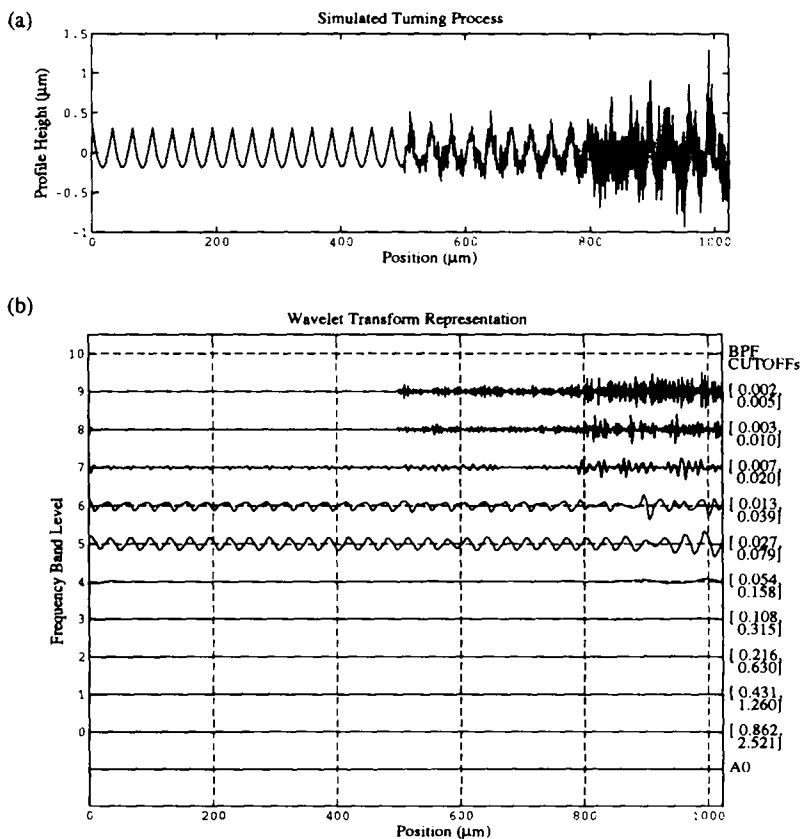


Figure 7 In-process monitoring by wavelet analysis. (a) A simulated turned (b) Wavelet transform representation.

higher resolutions are very sensitive to the noise condition of the process and hence can be a good indicator of the dynamic aspect of the process.

Mallat [12] proved that local maxima of the wavelet

transform modulus detect the locations of singularities of the function. This leads us to consider the possible application of wavelet transform to analyzing surfaces that have sharp scratches or peaks. Plateau honing is a typical process that

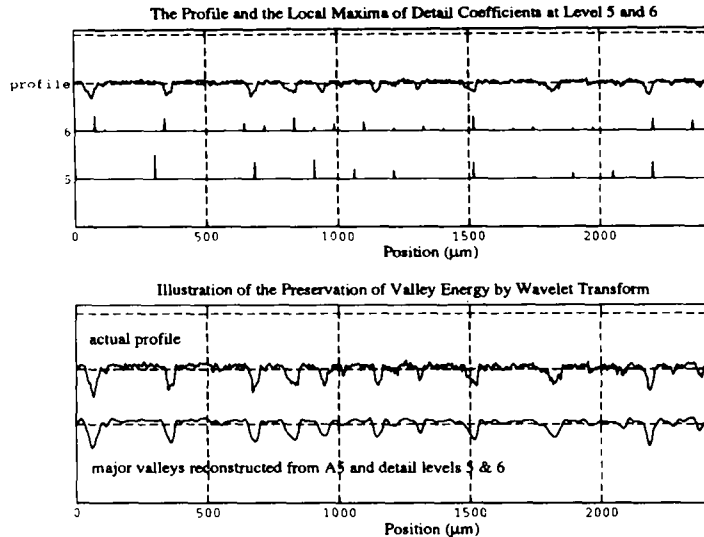


Figure 8 The application of wavelet transform to valley detection and representation. (a) Original plateau honed surface profile and its wavelet transform representation at resolution level 5 and 6. (b) A comparison showing the capability of wavelet transform for reserving abrupt features such as the valleys.

produces surfaces with both valley and plateau features. Based on the theory by Mallat, we expect that the position and width of the valleys of the surface can be easily revealed by wavelet transform of the profile, and this will help in characterizing such features as valley distribution, valley depth, valley widths, etc., which are considered important for the oil consumption and service life of engine. Figure 8 shows how good the wavelet transform is in detecting and preserving features related to valleys. The level at which the valleys are accurately located by the local maxima WT coefficients of that level has something to do with the width and depths of the valleys that are of concern. In practical situations, a threshold can be set for the amplitude of coefficients that can be included in the local maxima for detecting valleys of the specific size involved.

IV. CONCLUSION

From the theoretical review and practical application examples given above, It is clear that wavelet analysis is a valuable signal processing tool for linking the multi-scale features of an engineering surface with both its manufacturing and functional aspects. The multi-scale, or multi-resolution view, and space-frequency localization of signal components are the two essential properties of the wavelet transform that will benefit surface metrology. Specifically, multi-scale decomposition of surface structures provides a significant amount of information that may be traced back to the process, or utilized to predict the function. Simultaneously, the dynamic evolution and local events involved in the surface can be monitored and detected using the space-frequency localization of the wavelet bases. These concepts have been demonstrated as promising for applications in the area of functional filtering, process monitoring, and functional analysis. Extension of the multi-scale concepts to the three-dimensional surface analysis can be found in [7], where the capability of two-dimensional wavelet representation for pattern analysis of surface features such as lay, defects, and steps.

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