Int. J. Mach. Tools Manufact. Vol. 32, No. 1/2, pp. 171–177, 1992.
Printed in Great Britain

0890–6955/92 $5.00 + .00
Pergamon Press plc

ANALYSIS OF LAY CHARACTERISTICS OF THREE-DIMENSIONAL SURFACE MAPS

B. D. Boudreau* and J. Raja
Mechanical Engineering & Engineering Sciences Department
University of North Carolina at Charlotte

ABSTRACT:

This paper describes a technique which can characterize the surface lay structure contained within a three-dimensional surface map. The technique is based on a generally known fact, that two, closely spaced, parallel profiles will appear shifted whenever the dominant lay direction is not perpendicular to the profile direction. The cross-correlation function is utilized to evaluate these apparent shifts. It will be shown that the functional relationship of these shifts is a characteristic of the lay pattern. Simple linear models are then applied to the shifts to quantify the lay characteristics. The resulting models can be used to extract lay angles and curvatures which correlate directly to the manufacturing process used to create the surface.

1.0 INTRODUCTION

In recent years there has been an increasing interest in analyzing surface texture in all of its dimensions. This interest is primarily due to a desire by manufacturers to produce low cost products of superior quality. Quality however encompasses the appearance, durability and reliable functioning of the product. All three of these attributes, however, are affected in some fashion by surface properties, and for this reason, the control and verification of these surfaces has become a crucial part of the manufacturing process. A surface is a highly complex structure which is usually considered to consist of many simpler components. Figure 1 demonstrates this division, by showing the individual components which comprise the total surface structure.

Review papers by Whitehouse et al [1], Radhakrishnan [2], and Stout et al [3] present a large variety of two- and three-dimensional techniques available for the evaluation of surface roughness. However, few evaluation techniques are available which characterize surface lay. Peklenik and Kubo [4,5] describe techniques which evaluate the isotropic characteristics of three-dimensional surfaces. The first technique utilizes the peak value of the cross-correlation function for two parallel profiles of known separation. Observing the behavior of the peak value as a function of profile spacing indicates the relative strength of the surface lay. A second technique plots auto-correlation contours of radially traced surface profiles. A study performed by Tanimura et al [6] using this technique found that the shape of the resulting contours had features which were characteristic to the manufacturing process.

Three-dimensional surface maps are typically obtained by collecting a large number of evenly spaced two-dimensional profiles. For the most part, these data maps have not been used to their full potential. The evaluation and interpretation of these surface maps has been limited to techniques and parameters which bear a high degree of similarity to two-dimensional surface analysis. However, the three-dimensional surface map also contains a complete geometrical description of the surface structure. The most prominent and functionally important component of this structure is the surface lay (figure 1). The technique outlined below attempts to quantify the spatial characteristics of the dominant lay pattern.

2.0 BASIC CONCEPTS OF LAY CHARACTERIZATION

Relative and Absolute Shifts:

Three-dimensional surface maps are assumed to be totally representative of the surface from which they were collected. This basic premise indicates that all rudimentary surface features (roughness, waviness, ...) including lay are captured in the surface map. Since this premise is generally true, the main question now becomes: How do the

*) Visiting Scientist, Michigan Technological University, Houghton, Michigan.

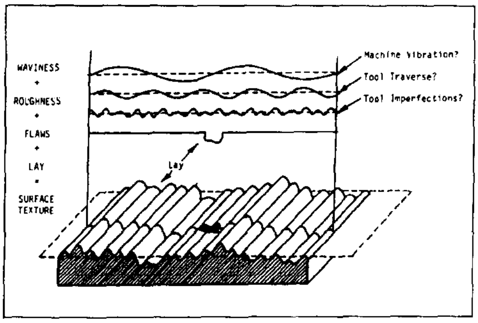


Figure 1 - Illustration showing the primary components of surface texture.

characteristics of surface lay manifest themselves in the surface map? The answer to this question is found by examining the nature of three-dimensional surface maps.

Three-dimensional maps are typically built up by collecting a large number of closely spaced parallel profiles using a stylus instrument. Figure 2 shows the relative position of three such profiles on curved and straight lay surfaces. The solid lines in this figure outline the top of the ridges which make up the surface lay. By examining the placement of the peak values in two adjacent profiles, it can be seen that they are shifted in position. From this it can be asserted that the surface lay will manifest itself as a shift between any two parallel profiles. Furthermore, this shift will always occur between two closely spaced profiles whenever the surface lay is not perfectly perpendicular to the profile direction. This shift is called a relative shift ($\delta\tau_i$) and its magnitude is directly related to the directional characteristics of the lay between the two profiles. It will be shown later that the magnitude of the relative shifts is a characteristic of the surface lay and that the relative shifts are all that is required to completely characterize the surface lay.

The absolute shift is another useful indicator of profile position. This shift is a measure of how far a particular profile is shifted relative to a reference profile, which is usually an edge (first) profile that is assumed to have zero shift. The absolute shift (ΔT_i) of any arbitrary profile can be evaluated by summing all of the relative shifts between it and the reference profile (i.e. $\Delta T_3 = \delta\tau_1+\delta\tau_2+\delta\tau_3$). The significance of the absolute shift is that it outlines the characteristic shape of the lay pattern when plotted.

Evaluating the Relative Shifts:

The cross-correlation function is ideally suited for evaluating the relative shift between profiles. This function is easily implemented in digital form as:

$$\rho_{xy}(\tau) = \frac{R_{xy}(\tau)}{\sqrt{\sigma_x^2}*\sqrt{\sigma_y^2}} \qquad (1)$$

where:

$$R_{xy}(\tau) = \frac{1}{N-\tau}\sum_{n=1}^{N-\tau} X_n Y_{n+\tau}$$

N=number of ordinates
τ=lag in number of ordinates

and

$$-1 \le \rho_{xy}(\tau) \le 1$$

and provides an index ($\rho_{xy}(\tau)$) which expresses the degree of similarity between two profiles as a function of a shift value τ. The peak value of the cross-correlation function indicates the point at which profiles X and Y are most similar, and the shift (τ) at which this occurs is the relative shift between the two profiles. The use of this function assumes that the lay characteristics are sufficiently strong such that the two adjacent profiles have a high degree of similarity.

The relative shifts between adjacent profiles are obtained by repeatedly applying the cross-correlation function to adjacent pairs of profiles. The results of applying this technique to three straight lay (0°, 30° and 45° lay angles) surfaces are shown in figure 4, while figure 5 shows the relative shifts for three different curved lay surfaces (1/4, 1/2 and 3/4 inch diameters). The lay tracing capabilities of the absolute shifts is demonstrated in figure 6.

Modeling the Lay Characteristics:

The characteristics of the relative shifts, shown in figures 4 and 5, suggest that a linear regression model can be used, but a model which is capable of representing the shifts from both straight and curved lay surfaces is needed. The

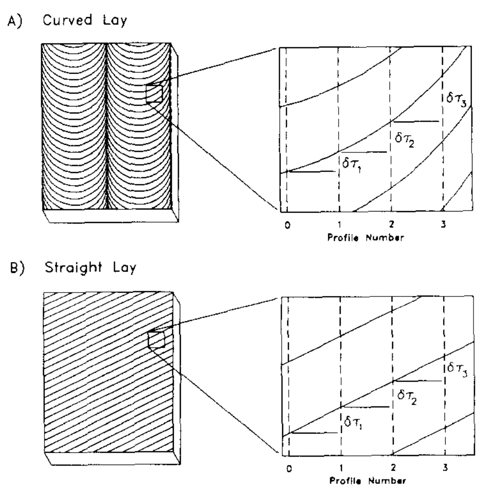


Figure 2 - Example surfaces showing the positional characteristics of relative shifts ($\delta\tau_i$) for parallel traced profiles (---).

required model can be determined by examining the relative shift ($\delta\tau_i$) characteristics for ideal curved and straight lay patterns, figure 2. This simplified set of lay patterns results from the fact that all surface lays can be represented by various combinations of three basic patterns: curved, straight and random, figure 3. The random lay pattern has not been considered because it leads to random relative shifts. Examining the ideal straight lay pattern in figure 2, it is seen that the relative shift between adjacent profiles remains constant throughout the entire data map. However, a close examination of the curved lay pattern shows that the relative shifts vary with respect to position within the data map. To resolve this case two assumptions must be made. The first of which is that the relative shift between two profiles is directly proportional to the slope of the lay direction between the profiles. This can be seen by considering the first order finite difference formula for the first derivative of the ideal lay pattern y(x).

$$\frac{dy}{dx} \approx \frac{Y_1-Y_2}{\Delta X} \approx \frac{\textit{Relative Shift}}{\textit{Profile Spacing}} \tag{2}$$

$$\frac{dy}{dx} \propto \textit{Relative Shift}$$

The second assumption is that the ideal curved lay can be described by a parabolic (second order) equation:

$$y_c = k(x-a)^2+b \tag{3}$$

where a and b are the x and y location of the inflection point. The parabolic model will be adequate if the lay curvature is smooth and continuous over the small range of the data map, but it was shown (equation 2) that the relative shifts are directly related to the slope of this function.

$$\frac{dy_c}{dx} = 2k(X-a) = \beta_0+\beta_1 x \tag{4}$$

This result clearly shows that the relative shifts for a curved lay surface can be described by a linear (first order) model. Similarly, the constant relative shifts of the straight lay surface can be described using a similar model. The similarities and distinctions between the two relative shift models can be illustrated by examining the equations characterizing the straight lay pattern.

$$y_s = c+dx \tag{5}$$

$$\frac{dy_s}{dx} = d = \beta_0 \tag{6}$$

A simple comparison of equations 4 and 6 clearly show that the slope of the modeled relative shifts will be significant for the curved lay pattern while being insignificant for straight lay. The slope of the model will never be zero but its significance can be determined using a simple statistical test (F-test). The underlying linear relationships of the relative shifts for curved and straight lay surfaces can be seen in figures 4 and 5, while the regression model parameters are listed in table 1. The results of applying the linear model to the absolute shifts are also included in table 1.

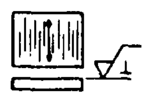
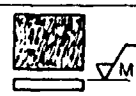
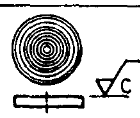
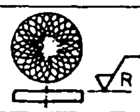
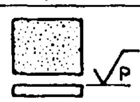
Lay Symbol	Meaning	Example Showing Direction of Tool Marks
=	Lay approximately parallel to the line representing the surface to which the symbol is applied.	
⊥	Lay approximately perpendicular to the line representing the surface to which the symbol is applied.	
X	Lay angular in both directions to line representing the surface to which the symbol is applied.	
M	Lay multidirectional.	
C	Lay approximately circular relative to the center of the surface to which the symbol is applied.	
R	Lay approximately radial relative to the center of the surface to which the symbol is applied.	
P[3]	Lay particulate, non-directional, or protuberant.	

Figure 3 - Examples of typical lay patterns.

3.0 EXTRACTION OF MANUFACTURING PARAMETERS

A study by Raja [7] considers a manufactured component to be a "fingerprint" of the process that produced it. The evaluation of manufacturing parameters from three-dimensional maps will, therefore, be used to illustrate the capabilities of the lay characterization process. There are two distinct methods to extract manufacturing parameters (lay characteristics) from the linear and parabolic models fitted to the absolute and relative shifts. The first method uses the relative shift model parameters to obtain closed form equations for the lay angle and curvature. The second method obtains the lay angle and curvature from the absolute shifts. This method, however, requires the use of the fitted models and finite difference techniques.

Relative Shift:

The closed form solutions are obtained by equating the coefficients of the relative shift model,

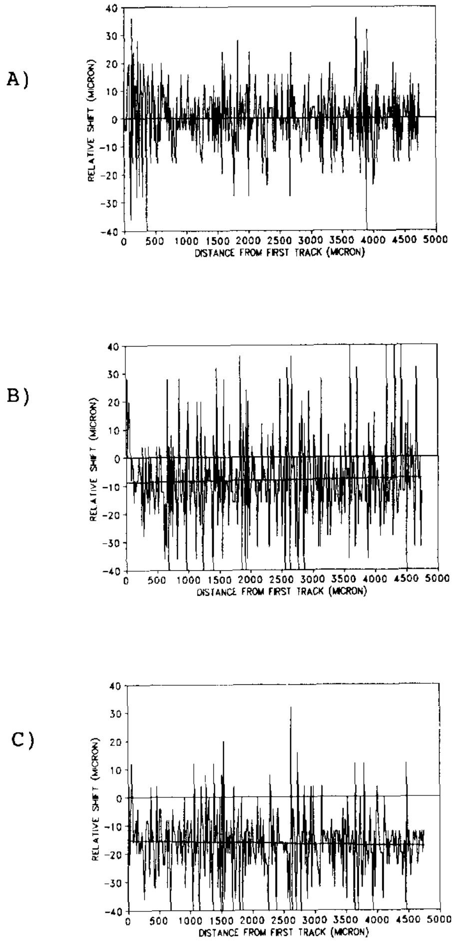


Figure 4 - Examples of the actual (~~~) and modeled (——) relative shifts for a ground surface orientated at A) 0, B) 30, and C) 45 degrees to the track collection direction.

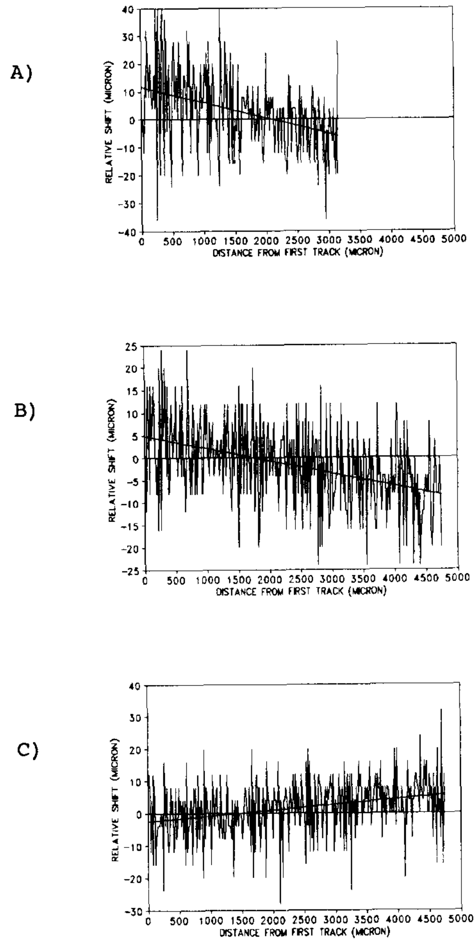


Figure 5 - Examples of the actual (~~~) and modeled (——) relative shifts for A) 1/4, B) 1/2 and C) 3/4 inch milling cutters.

$$\delta\tau_i = \beta_0 + \beta_1 x_i \qquad (7)$$

and the models describing the slope of the ideal lay patterns given as equations 4 and 5. However, before the comparison can be performed, the relative shift model has to be converted to proper units. This conversion is performed using equation 2, which gives the resulting relationship,

$$\frac{dy}{dx} = \frac{\delta\tau_i}{\Delta X} = \frac{\beta_0}{\Delta X} + \frac{\beta_1}{\Delta X} x_i \qquad (8)$$

where ΔX is the spacing between the parallel profiles.

Equating the coefficients of equations 6 and 8 gives an expression which describes the orientation of the surface lay:

$$d = \frac{\beta_0}{\Delta X} \qquad (9)$$

where d is the slope of the original ideal straight lay model (equation 5). This expression can be converted to an angle by taking the arc tangent of each side.

$$\theta = \tan^{-1} d = \tan^{-1} \frac{\beta_0}{\Delta X} \qquad (10)$$

The lay angle (θ) in this equation is measured from a line perpendicular to the track direction. In other words a lay perpendicular to the profile direction would have a zero lay angle. The lay angle calculated will have a clear physical interpretation only when the slope of the relative shift model is insignificant.

Equating the coefficients of equations 4 and 8 yields equations which define the ideal curved lay model (parabolic) parameters in terms of the relative shift model parameters. Performing the required operations and simplifying results in the following equations,

where a and k are two of the original

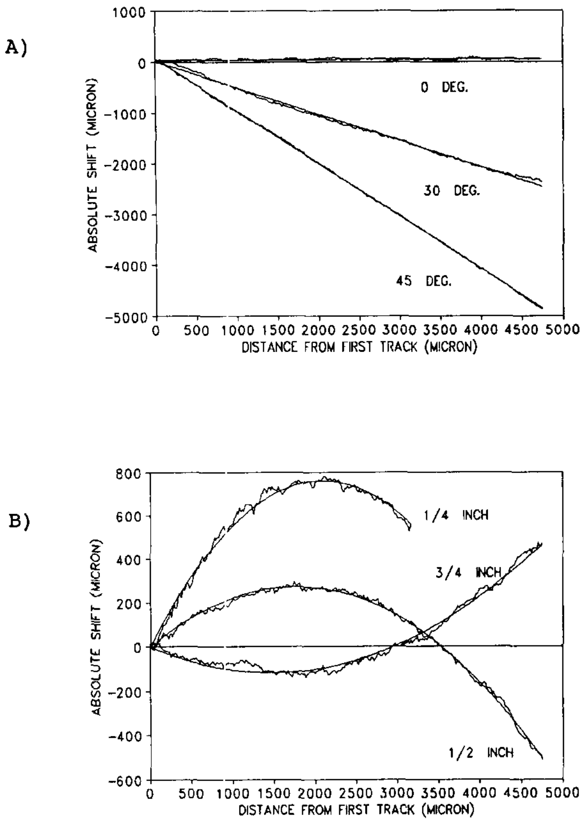


Figure 6 - Actual (····) and modeled (——) absolute shifts for A) straight and B) curved lay surfaces.

$$k = \frac{\beta_1}{2\,\Delta X} \quad (11)$$

$$a = -\frac{\beta_0}{\beta_1} \quad (12)$$

ideal curve lay model parameters. The remaining model parameter b must be obtained directly from the original model (equation 3) using the boundary condition that the reference track has zero shift ($y_c(0) = 0$). Substituting in the appropriate terms and simplifying gives:

$$b = -\frac{\beta_0^2}{2\,\beta_1\,\Delta X} \quad (13)$$

An equation expressing the lay curvature in terms of the regression model parameters can now be obtained by using the standard curvature formula,

$$\frac{1}{\rho} = \frac{y''}{[(y')^2+1]^{3/2}} \quad (14)$$

where ρ is the radius of curvature, y' and y'' are the first and second derivatives respectively. Obtaining the necessary derivatives of equation 3 and substituting gives,

$$\frac{1}{\rho} = \frac{2k}{[(2k(x-a))^2+1]^{3/2}} \quad (15)$$

This equation expresses the curvature as a function of position within the data map. An estimate of the radius of curvature can be obtained by evaluating this function at x=a. This estimate can be justified by considering that the inflection point of the original model is at the point x=a, and this is the region where the parabolic assumption would be most appropriate. Thus, equation 15 can be reduced to,

$$\rho = \frac{\Delta X}{|\beta_1|} \quad (16)$$

which is a simple formula relating radius of curvature to the slope of the relative shift model and the profile spacing. Note, the absolute value of the slope is taken to eliminate the possibility of negative curvature values. This equation is only valid if the statistical test indicates that the slope of the relative shift model is significant.

Absolute Shift:

This approach is based on the assumption that the absolute shifts trace the characteristic lay pattern. However, due to the difference between straight and curved lay patterns, two different evaluation techniques are required. The severity of this problem is limited because the presence of curvature is easily detected by the significance of the slope of the linear relative shift model.

Due to the linear nature of the straight lay the evaluation of the lay angle can be easily performed by applying a linear regression model to the absolute shifts. The lay angle is obtained from this model by taking the arc tangent of the slope. As stated previously, the lay angle is physically significant only if the slope of the relative shift model is zero (straight lay). If the slope is not zero, the lay angle would only indicate the general direction of the curved lay.

The radius of curvature is evaluated by using the standard curvature formula (equation 14). The required derivatives are evaluated through the use of finite difference formulas. Using finite difference formulas allows for the evaluation of the curvature at each point over the entire range of the absolute shifts. The individual curvature values are then averaged to obtain the final curvature value. An additional step which improves the curvature estimate is to use the modeled absolute shifts. Modeled absolute shifts can be easily generated by summing the modeled relative shifts evaluated at each profile or using the parabolic curved lay model. Using modeled absolute shifts improves the estimate by eliminating the large fluctuations present in the original shifts.

4.0 DISCUSSION OF RESULTS AND CONCLUSIONS

The complete lay characterization results for the six straight (ground) and curved (milled) surfaces are listed in table 1. The relative and absolute shift models are also graphically depicted in

	Orientation or Diameter	Relative Shift Intercept	Slope	F-test	Estimated Orientation or Diameter	Absolute Shift Intercept	Slope	F-test	Estimated Orientation or Diameter
GROUND	0°	0.660	-1.949×10^{-4}	No Curvature	2.362°	44.61	3.588×10^{-3}	Lay Angle	0.206°
GROUND	30°	-8.670	3.251×10^{-4}	No Curvature	-28.64°	18.10	-5.209×10^{-1}	Lay Angle	-27.52°
GROUND	45°	-15.29	-4.232×10^{-4}	No Curvature	-43.92°	78.49	-1.038	Lay Angle	-46.08°
MILLED	0.25 in.	11.63	-5.566×10^{-3}	Curvature	0.230	294.3	1.743×10^{-1}	Lay Angle	0.274
MILLED	0.50 in.	4.940	-2.795×10^{-3}	Curvature	0.458	325.0	-1.055×10^{-1}	Lay Angle	0.472
MILLED	0.75 in.	-2.550	1.732×10^{-3}	Curvature	0.739	-195.9	9.571×10^{-2}	Lay Angle	0.744

Table 1 – Linear regression model parameter values for straight and curved lay surfaces. The estimated lay angle and curvature are also given for each surface. A track spacing of 16 micron was used for all calculations.

figures 4 and 5. An examination of these figure 4 shows that an increase in the lay angle produces a vertical offset while a smaller radius of curvature increases the slope of the relative shifts. Both of these observations are in agreement with the lay angle (10) and radius (16) equations. The significance test (F-test) of the slope is clearly able to detect the presence of lay curvature. A comparison of the absolute shift models to the actual data (figure 6) shows the high degree of correlation for both the linear (straight lay) and the parabolic models (curved lay).

The extraction of manufacturing parameters also provides encouraging results. The calculated lay angles listed in table 1 show that the lay angles were estimated to within 2.5° of the actual value. Similarly, the diameter of the milling cutters were estimated to within 0.050 inches. The results given in table 1 indicate that the lay angle and curvature can be evaluated from either the relative or absolute shifts, whichever is more convenient.

The results listed in table 1 illustrate the capabilities of the lay characterization technique, but the models it produces also have other uses. In a previous study [8,9] it was found that the relative shift model could be used to reduce track alignment errors within three-dimensional surface maps. The error

Diameter	k	a	b	Zero Crossing
0.25	-173.9×10^{-6}	2089	759.4	4178
0.50	-87.36×10^{-6}	1767	272.8	3534
0.75	54.12×10^{-6}	1472	-117.3	2944

Table 2 - Calculated coefficients for the assumed parabolic model. A track spacing of 16 micron was used for all calculations.

reduction technique turned out to be highly robust, even in the presence of large amounts of noise. In the initial development stages of the lay characterization technique it was also observed that the models were sensitive to orthogonality errors within the data collection system. This sensitivity was detected through the characterization of random lay (EDM) surfaces. When the results from these surfaces were examined they indicated that a straight lay was present. These erroneous results persisted for different random surfaces and were later attributed to the non-orthogonality between the X and Y stages of the surface profilometer. This observation also points out that steps must be taken to eliminate orthogonality errors before valid lay characterization results can be obtained.

References:

1. D. J. Whitehouse and P. Vanherck, Survey of Reference Line in the Assessment of Surface Texture, Ann. CIRP, 21 (2) (1972) 267-273.
2. V. Radhakrishnan, Analysis of Some Reference Lines Used for Measuring Surface Roughness, Proc. Instn. Engrs.,187 (1973) 575-582.
3. K. J. Stout and P. J. Sullivan, The Analysis of the Three-Dimensional Topography of the Grinding Process, Ann. CIRP, 38 (1) (1989) 545-548.
4. J. Peklenik and M. Kubo, A Basic Study of a Three-Dimensional Assessment of the Surface Generated in a Manufacturing Process, Ann. CIRP, 16 (1968) 257-265.
5. M. Kubo and J. Peklenik, An Analysis of Micro-Geometrical Isotropy for Random Surfaces, Ann. CIRP, 16 (1968) 235-242.
6. Y. Tanimura, E. C. Teague, F. E. Scire, R. D. Young and T. V. Vorburger, Graphical Signatures for Manufactured Surfaces, J. Lubr. Technol., 104 (1982) 533-537.

7. J. Raja, Field Testing of Machine Tool Diagnostic Techniques Using Surface Metrology, Ann. CIRP, 32 (1) (1983) 503-506.
8. B. D. Boudreau, Analysis of Three-Dimensional Surface Texture, M. S. Thesis, Michigan Technological University, 1989.
9. B. D. Boudreau and J. Raja, A Study of Two- and Three-Dimensional Surface Characterization Techniques, Submitted to Precision Engineering.