Sampling Methods for Circles in Coordinate Measuring Machines

O. Odyappan, J. Raja, R. J. Hocken, U. Babu and K. Chen
Precision Engineering Laboratory
University of North Carolina at Charlotte
Charlotte NC

1.0 Introduction

The increasing need for tighter tolerances, emphasis on process control to achieve quality in manufacturing, and the trend towards automation has resulted in the need for fast and accurate dimensional and form measurement systems. Co-ordinate measuring machines (CMMs) have been designed to satisfy the measurement needs of modern manufacturing. As a result, in recent years CMMs have become the instrument of choice for dimensional measurement. CMMs provide detailed dimensional data on most machined part surfaces. They are also capable of determining the relationship between various part features. Since a coordinate measuring machine acquires data on a point-by-point basis, the results obtained from an inspection need to be analyzed to create a geometric model of the feature or features being measured (also referred to as "substitute geometry"). This analysis is handled by algorithms whose results are strongly dependent upon many factors, including the sampling method chosen for data acquisition, the noise in the data, systematic deviations of the measured surface from the ideal, systematic and random errors in the measurement and algorithm correctness[1,2]. Therefore, though it may appear to the user that data of high accuracy are obtained, the results reported after analysis may contain large errors.

The overall measuring uncertainty of a CMM depends on the hardware, consisting of the machine and the probe, the number and distribution of discrete points taken on the surface, and the software used to process the data. There are Standards for testing the performance of CMMs that describe tests to obtain information on machine accuracy and repeatability. Standards for checking the algorithms used for establishing substitute geometry are being developed [3]. In the area of sampling methods (number of points and their distribution) the British Standard BS 7172 addresses the problem for the most important geometric elements[4]. Their recommendations is shown in Table 1. Although information is available in the areas that contribute to the uncertainty in CMM measurement, a systematic investigation to study and to characterize the uncertainty due to different sampling methods has not been done yet. The results from such a study can be used to develop sampling methods for different geometric elements in the presence of systematic and random errors on both the part and the CMMs.

This paper presents the results of an investigation of the influence of the measuring system, sampling methods and algorithms on the inspection results of circles in CMMs. The systematic biases in the results, due to the algorithms chosen for analysis are also identified. The parameters that affect the measurement process and the different algorithms used for computing the substitute geometry are first analyzed. Sampling methods for circles in CMMs are then investigated using computer simulation. The computer simulation and recommended strategies

Element	Minimum Number of Points Mathematical	Recommended	Comments Regarding Minimum Number of Points
Line	2	5	
Plane	3	9	Approximately three lines of three points.
Circle	3	7	To detect upto six lobes.
Sphere	4	9	Aproximately three circles of three in parallel planes.
Cylinder	5	12	Circles in four planes for information on straightness.
		15	Five points on each circle for information on roundness.
Cone	6	12	Circles in four planes for information on straightness.
		15	Five points on each circle for information on roundness.

are validated by measuring parts in a CMM. The following sections describe the details of the investigation.

2.0 Parameter Identification

One of the most important task in establishing a viable measurement strategy is to identify the parameters that affect the measurement process and the measuring system. The various parameters that could influence the measurement strategy of a circular profile are given in Figure 1. These parameters may be grouped in to the following categories:

A. Properties of the part
    1. those that relate to geometry and tolerance
    2. manufacturing process.

B. Measurement function
    1. purpose of measurement
    2. required confidence in the measurement.

C. Properties of the measurement system
    1. systematic errors
    2. random errors.

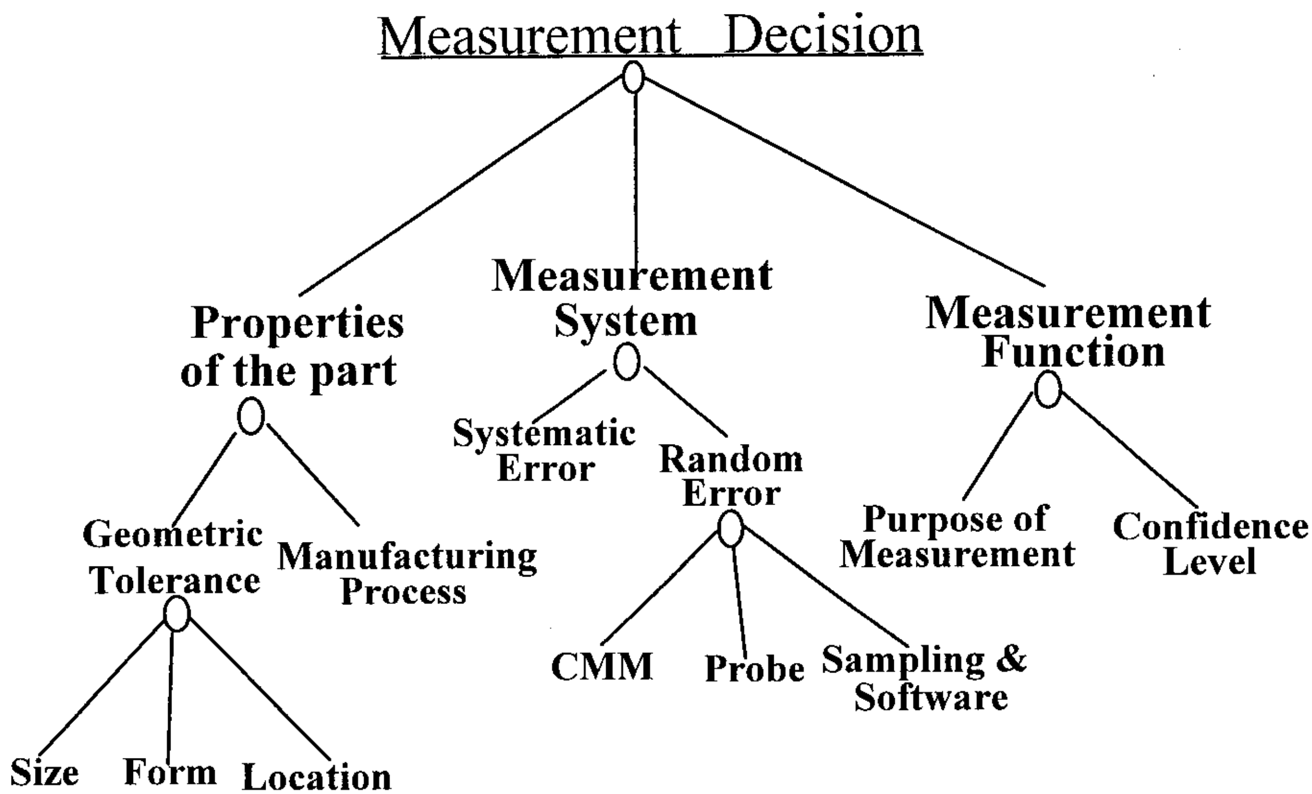
2.1 Properties of the Part

The choice of algorithm and sampling strategy is affected by the properties of the part such as geometry, form errors, characteristics of the process used to manufacture the part, tolerance specifications, etc. These part characteristics can be classified into those relating to the geometric dimensioning and tolerancing of the part and those of the manufacturing process.

Geometry and Tolerance:

The initial assumption was that the most relevant parameters would be size, size tolerance, form tolerance, surface finish and entity type (internal or external). Our experiments showed, however, such features as the size of the part has no effect on the measurement strategy as long as the ratio of the form error to the part size is the same. The measurement depends only on the shape of the skin (i. e., the outer layer) of the part, which reflects the form error.

However, the size of the feature and amplitude of the form error must be considered when selecting the probe to be used for measurement in order to avoid filtering effects. Especially for a small part in which the number of undulations per unit length is high, the probe diameter should be carefully selected so that it is small enough to reach the bottom of any undulation. This limitation is not particularly restrictive. The maximum diameter of the probe that may be used can be estimated by computing the radius of curvature at the bottom of the sine wave of a given amplitude. It is found that the maximum radius of the probe that can be used for measurement is a function of the part radius, number of lobes, and the lobe amplitude, and is given as,


Measurement Decision
Properties of the part
Measurement System
Measurement Function
Geometric Tolerance
Manufacturing Process
Systematic Error
Random Error
Purpose of Measurement
Confidence Level
Size
Form
Location
CMM
Probe
Sampling & Software

$$d_{max} = \frac{1}{2A}\left(\frac{D}{N}\right)^2$$

where d_{max} is the maximum diameter of the probe capable of reaching the bottom of a lobe of amplitude, A, in a part of diameter, D, with N lobes. In the presence of form error the sampling strategy must be carefully selected when position and form error are to be evaluated. For example, four points distributed uniformly on three lobed part will introduce error in the location of the center and six points on a three lobed part could fail to detect the form error.

Manufacturing Process:

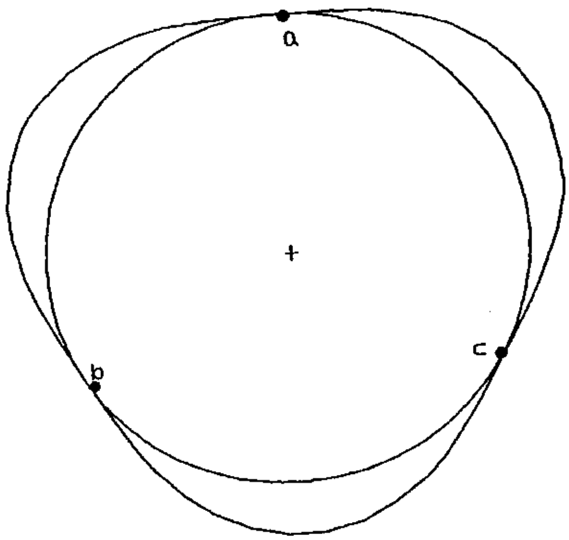
In general manufacturing process leaves a finger print on the surface of the part. These fingerprints are often in the form of geometric irregularities which are often classified according to the frequencies. Typically, high frequency components are called roughness and lower frequency component as waviness. As a result of our literature search and survey [5] it is found that for most manufacturing process, the dominant lobing is low order (2- 7) but rarely higher. The lobe amplitude, in general, drops of dramatically with the number of undulations per revolution. In this study only low frequency effects such as lobing is included. The probe used in a co-ordinate measuring machine filters most of the high frequency component, making the low frequency or the lobing effect as the predominant manufacturing process effect.

2.2 Measurement Function

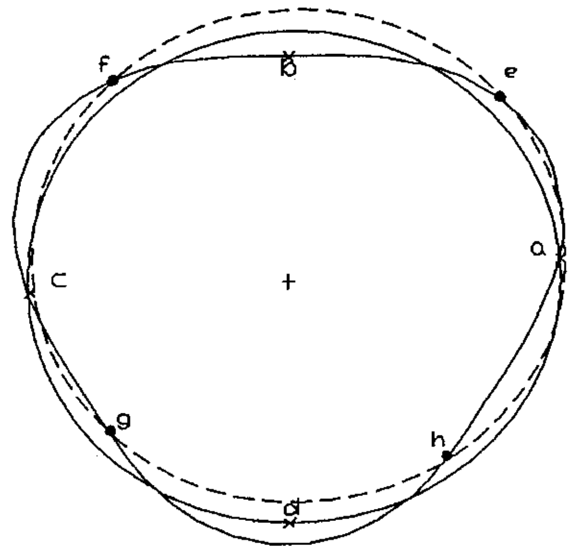
The function of the measurement affects both the algorithm and the sampling strategy. The purpose of the measurement and the required confidence are the two important aspects in the measurement function.

Purpose of Measurement:

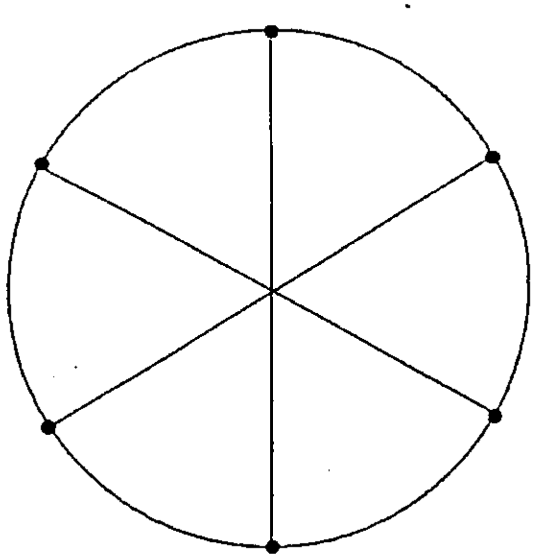
The purpose of the measurement affects both the choice of the algorithm and the sampling strategy. If the purpose is tolerance verification, then the first step in algorithm selection is to determine which algorithm is called for in the ANSI Y14.5 Standard or the equivalent ISO standard [6,7], one would choose the maximum inscribed (circle and cylinder) for the size of an internal feature, and the minimum circumscribed (circle or cylinder) for the size of an external feature. The form of the same feature would be determined by the range of the residuals using the mini-max algorithm, and the location would be established by the center of the circle obtained by the algorithm used to establish size. For the case of process control, the least-squares algorithms might always be the algorithm of choice, due to their stability and ease of computation. Once the correct algorithm is established, then the results will also be affected by sampling particularly when it is desired to use the minimum number of points. Consider the part shown in Figure 2., having the familiar three lobed pattern. If this part is sampled using three points, then it is readily seen, as illustrated, that not only is it clearly possible to obtain the wrong size, but also to completely fail to detect the form error. There is, therefore, uncertainty on the size measurement of twice the form error and the possibility of total uncertainty in the form error determination. A similar problem exits on the same part for



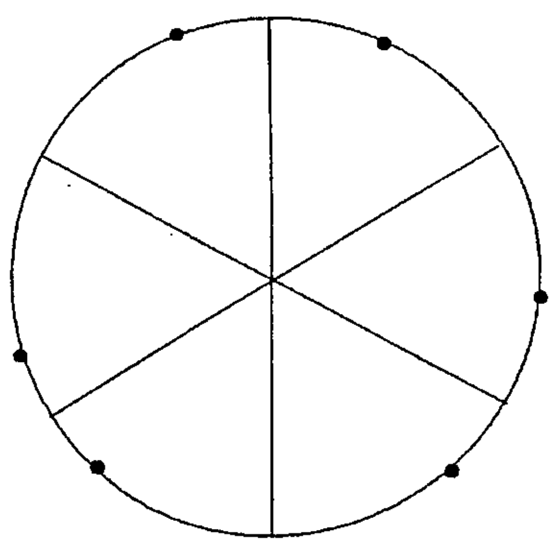
Polar Plot



Polar Plot



Equi-Distant



Stratified

location tolerance, when four points are used to measure its position. This is illustrated in Figure 3. The four-point sampling may give the correct radius, but the center can be offset from the true position. In Figure 3., sampling points a, b, c, and d give a different center than sampling points e, f, g, and h. Clearly in the presence of form error there are some sampling strategies that yield very little confidence regarding the final result.

2.3 Properties of the Measurement System

The measurement system itself produces uncertainties that affect the sampling strategy of choice. The parts of the system that affect the measurement results are the errors in geometry and motion of the coordinate measuring machine and its probe. For machines which are parametrically calibrated, detailed maps of the systematic errors can be developed and used either to correct the measurement results or to estimate how these errors will contribute to apparent errors of form. Most machines now have many of these systematic errors corrected in the machine software. The emphasis in this study is mainly on the residual systematics and any apparently random errors that are introduced by the machine probe interaction with the part.

Systematic Measuring Machine Errors:

Most CMMs are evaluated according to American National Standard for measuring machines, B89.1.12M -1991 [8]. As a result of this evaluation the machine user has four numbers which allow estimates of the residual machine systematics. They are the linear displacement accuracies of each axis and the ball bar performance. For the purpose of evaluating the effects of these residual systematics on the measurement of circular and cylindrical geometry, these numbers are not directly applicable. This is because the linear displacement accuracy is performed along lines through the work zone, and the ball bar performance measured at a wide variety of locations such that it would be extremely unlikely that the total spread in ball bar data would ever be reflected in the measurement on a cylindrical part. Although these systematics will contribute to apparent errors of form, they cannot be included in the analysis in any meaning full manner.

"Random" machine errors:

In addition to the systematic geometric errors, most geometric measuring machines exhibit apparently random errors. These errors are caused by interaction of the probing system with machine dynamics and lack of repeatability in the probing mechanism. In order to asses the "noise" of the measurement process, the ASME B89.1.12 probing performance test using the probe system and probe approach rates and distances which will be used in actual measurement must be conducted. One-half the value obtained for the working tolerance with that particular stylus in the point-to-point probing performance test be used as the estimate for the three sigma " noise" of the machine.

Although the parameters that affect the measurement process originate from different sources, it is clear that form error, random error (from the part and the measuring machine), sampling scheme and the algorithm used are the basic factors that affect the results. The effect of these basic factors were analyzed in detail in this investigation.

3.0 Sampling Strategies and Mathematical Methods

The number of methods that one can arrive at for measuring a circular or cylindrical part, using a coordinate measuring machine, is somewhat limited. Sampling theory is , of course well developed in electrical engineering[9]. In measuring on a CMM, however, due to limitations in speed, much more limited sampling is desirable. Further, because of software limitations, certain sampling strategies are difficult to implement. In this investigation, uniform sampling, random sampling, and stratified sampling has been used. Uniform sampling (also known as equi-distant or equi-angular) is where the surface to be measured is divided into uniform intervals, such as angles for a circle, and a point chosen at each of these intervals. Stratified sampling is a variation of uniform sampling, where within each of the uniform intervals a random point is selected within that interval. Figure 4. illustrates these two procedures.

3.1 Mathematical Methods

The parameterization of geometric elements used in coordinate metrology was investigated by the British Standards Institution, while a Standard on "Assessment of Position, Size and Departure from Nominal Form of Geometric Features" (BS 7172) [4] was developed. This Standard clearly outlines the parameterization for various geometric elements used in coordinate metrology. The geometric elements can be parameterized in different ways, and the Standard claims that the parameterization chosen has the following property:

> "Small changes in the geometric element usually result in correspondingly small changes in the parameter values".

Circles in a Specified Plane:

The parameterization, as outlined in BS 7172 [9] for a circle, is

"A circle C in the plane should be specified by its center, (x_0,y_0), and its radius, r.
NOTE 1. Any point (x,y) on C satisfies the equation:

$$(x-x_0)^2+(y-y_0)^2=r^2 \qquad (2)$$

NOTE 2. Numerical inaccuracies are likely to arise in the use of this parameterization for a circle related to a set of data points lying on or near an arc whose length is much smaller than its radius."

3.2 Best Fitting Geometric Element Computation

General Approach:

The general approach for developing substitute geometry for circles based on Least-squares (LS), Minimum Zone (MZ), Maximum Inscribed (MI) and Minimum Circumscribed (MC) criteria is discussed in this section. The two main requirements for solving the best fitting problem for any geometric element are proper parameterization and criteria for best fit. The parameterization has been addressed in the previous section, and the criteria is addressed in this section.

Consider a circle, C, with center (x_0,y_0) and radius r. Any point on the circle, P_i (x_i,y_i), satisfies the equation

$$(x_i - x_0)^2 + (y_i - y_0)^2 = r^2 \qquad (3)$$

Then the residual distance, d_i, (i.e., the radial distance of P_i from C) is

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} - r \qquad (4)$$

Hence, the problem is

$$\min\left[\frac{1}{n}\sum_{i=1}^{n} |d_i|^p\right]^{\frac{1}{p}} \qquad (5)$$

$$\Rightarrow \quad \min\sum_{i=1}^{n} |d_i|^p$$

where n is the number of data points. This is called the L_p norm estimation problem. The cases $p = 2$ and $p = \infty$ are of special interest in substitute geometry. When $p = 2$,

$$L_2 = \min\left[\frac{1}{n}\sum_{i=1}^{n} |d_i|^2\right]^{\frac{1}{2}} \qquad (6)$$

$$\Rightarrow \quad \min\sum_{i=1}^{n} d_i^2$$

$$\min\sum_{i=1}^{n}\left(\sqrt{(x_i-x_0)^2+(y_i-y_0)^2} - r\right)^2 \qquad (7)$$

This is called the least-squares problem. When $p = \infty$, the equation reduces to the L_∞ estimation problem to minimize the maximum of the absolute values of d_i

$$L_\infty = \min_{(x_0, y_0, r)} \ \max_{1 \le i \le n} |d_i| \qquad (8)$$

This is called the mini-max or the minimum zone or the Chebyshev problem [14, 15, 16, 17, 18].

The minimum circumscribed circle problem is to find the center, (x_0, y_0) and the radius, r, of the circle which minimizes r subject to

$$\sqrt{(x_i-x_0)^2+(y_i-y_0)^2} \le r \qquad for\ i=1,2,..n$$
$$r = \max_{1\le i\le n} \sqrt{(x_i-x_0)^2+(y_i-y_0)^2} \tag{9}$$

This problem can be written as the L_∞ norm estimation problem,

$$\min_{(x_0,y_0)} \max_{1\le i\le n} \sqrt{(x_i-x_0)^2+(y_i-y_0)^2} \tag{10}$$

$$H(x_0,y_0) = \max_{1\le i\le n} \sqrt{(x_i-x_0)^2+y_i-y_0)^2} \tag{11}$$

The Anderson-Osborne-Watson algorithm is used to solve this problem.

The maximum inscribed circle problem is to find the center and the radius, r, to maximize r subject to the constraints

$$\sqrt{(x_i-x_0)^2+(y_i-y_0)^2} \ge r \qquad for\ i=1,2,..n$$
$$r = \min_{1\le i\le n} \sqrt{(x_i-x_0)^2+(y_i-y_0)^2} \tag{12}$$

This equation can be written as the L_∞ norm estimation problem,

$$\max_{(x_0,y_0)} \min_{1\le i\le n} \sqrt{(x_i-x_0)^2+(y_i-y_0)^2} \tag{13}$$

This can be written as the minimization problem by subtracting it from a constant, C

$$\Rightarrow \qquad - \min_{(x_0,y_0)} \max_{1\le i\le n} \left(-\sqrt{(x_i-x_0)^2+(y_i-y_0)^2}\right) \tag{14}$$

$$\Rightarrow \qquad C - \min_{(x_0,y_0)} \max_{1\le i\le n} \left(C - \sqrt{(x_i-x_0)^2+(y_i-y_0)^2}\right) \tag{15}$$

Numerous algorithms have been proposed for solving least-squares and mini-max problems. We used the Gauss-Newton method [10] for solving the least-squares best fit problem. The Anderson-Osborne-Watson algorithm [11] was used for solving the mini-max, maximum inscribed and minimum circumscribed circle problem. These methods use first derivative information about the d_i's to determine the search direction. Figure 5. shows results from the four algorithms (LSC, MZC, MCC, MIC) for a three lobed part with 2.5 microns noise.

4.0 Computer Simulation of Measurements

In order to develop the best measurement strategy, numerous experiments had to be conducted to study the influence of different parameters on the measurement strategy. The core

Algorithm **:** ***Least Squares***

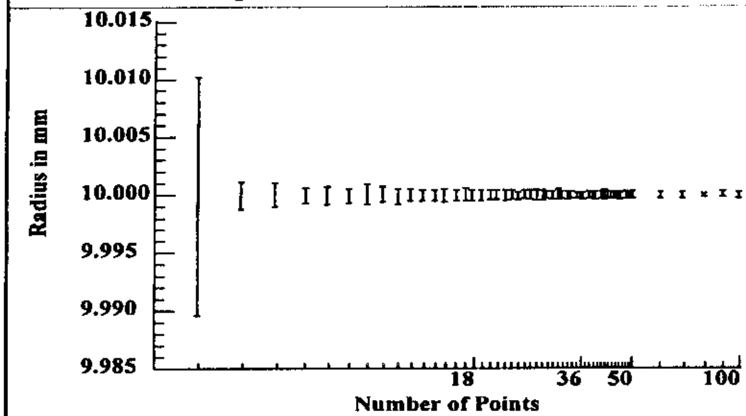
Radius **:** ***10.00 mm***

Random Noise **:** ***2.5 microns***

No. of Lobes/Lobe Amp **:** ***3/10 microns***

Sampling Scheme **:** ***Equi-Distant***

Total Repetitive Runs **:** ***100***



Algorithm **:** ***Minimum Zone***

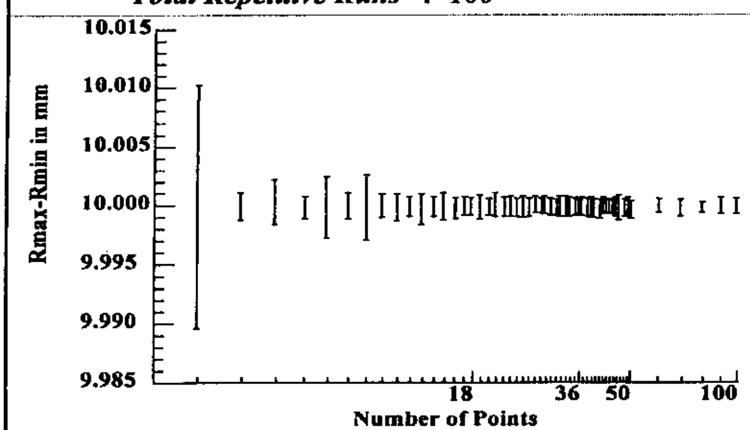
Radius **:** ***10.00 mm***

Random Noise **:** ***2.5 microns***

No. of Lobes/Lobe Amp **:** ***3/10 microns***

Sampling Scheme **:** ***Equi-Distant***

Total Repetitive Runs **:** ***100***



Algorithm : ***Minimum Circumscribed***

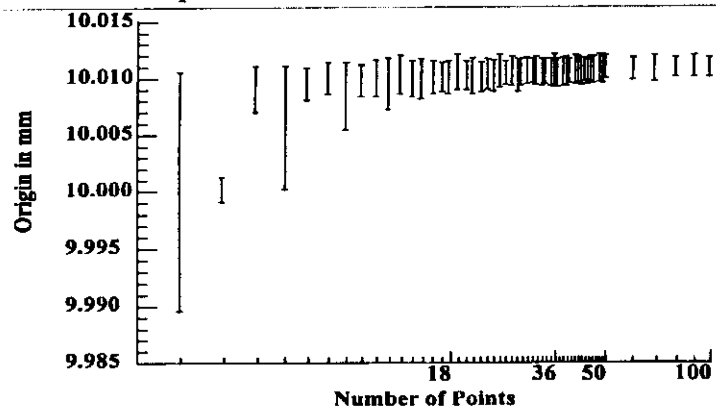
Radius : ***10.00 mm***

Random Noise : ***2.5 microns***

No. of Lobes/Lobe Amp : ***3/10 microns***

Sampling Scheme : ***Equi-Distant***

Total Repetitive Runs : ***100***



Algorithm : ***Maximum Inscribed***

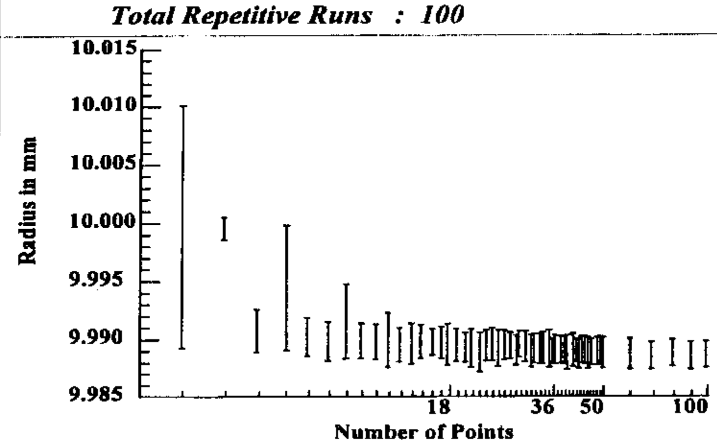
Radius : ***10.00 mm***

Random Noise : ***2.5 microns***

No. of Lobes/Lobe Amp : ***3/10microns***

Sampling Scheme : ***Equi-Distant***

Total Repetitive Runs : ***100***

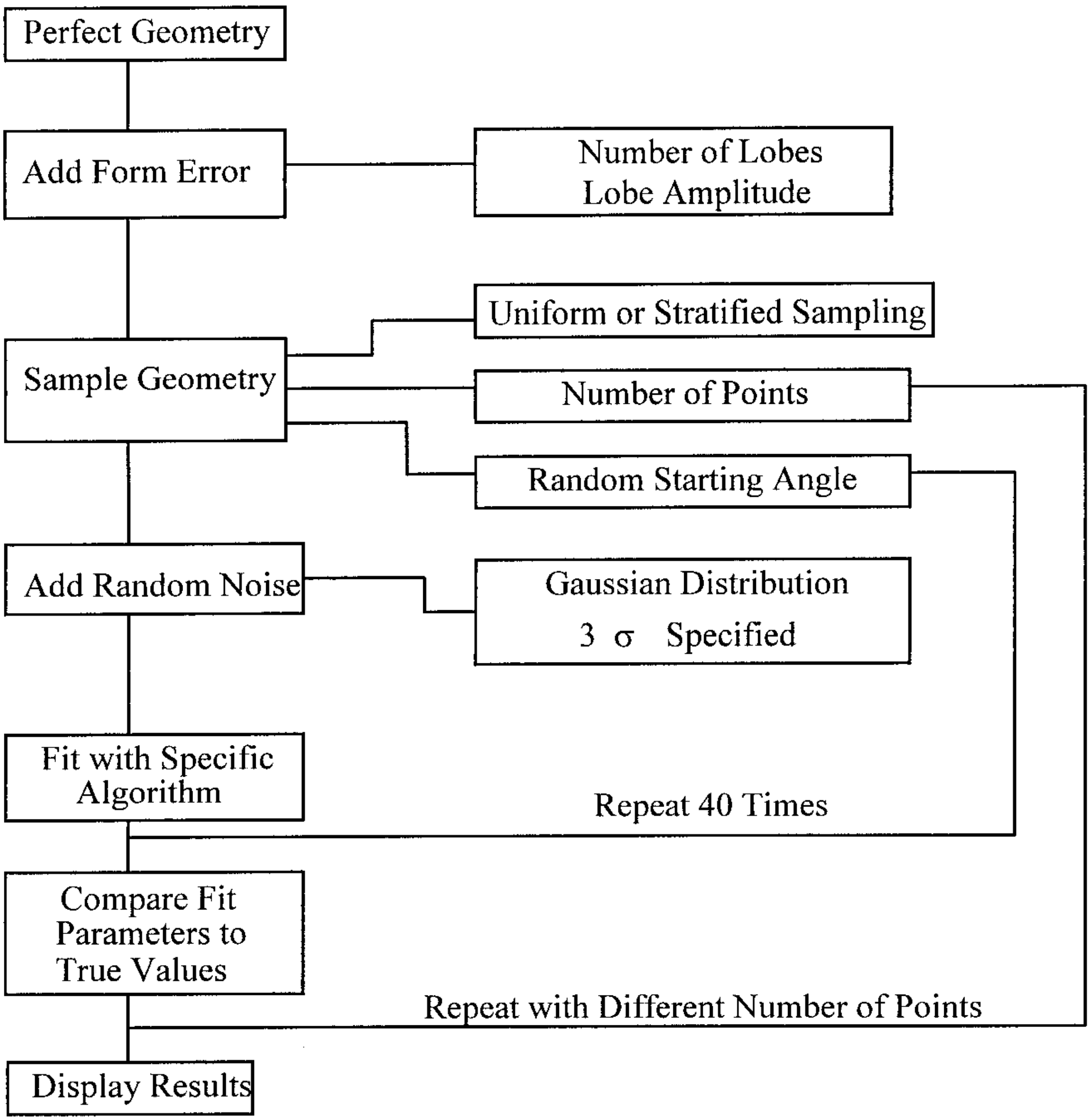


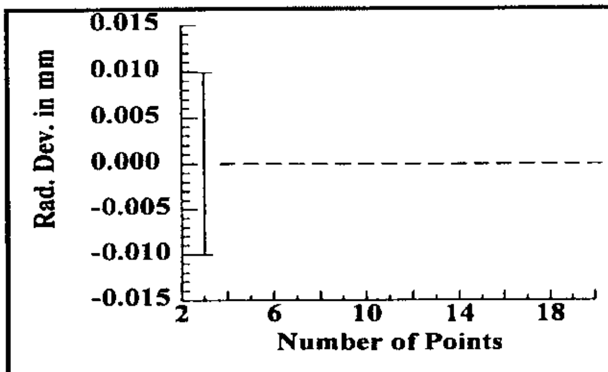
of our effort was directed around computer simulation of actual measurement practice, using geometries with different form errors, sampled with different sampling techniques, sampling densities, and random measurement errors. The procedure is shown schematically in Figure. 6. It begins by generating a perfect feature. Next, form error may be added to this feature. In the case of circles, lobes around the circumference have been added with variable lobe amplitude. Once the form error was added, the geometry was sampled using one of three techniques. The first technique was uniform sampling, the second technique stratified sampling, and the final technique random sampling. After several initial experiments, the random sampling was abandoned since it is difficult to do on a CMM and also yields unpredictable results. For different sampling strategies, different number of points were used. For a specific sample and a specific number of points, random noise was added to the coordinates to represent measurement uncertainty, and then the coordinates were fit using one of many algorithms. The results from the algorithm were compared to the "true" value and differences between the results and the "true"value displayed. For the purpose of these experiments, the "true" value was considered to be the value obtained using the selected algorithm on the selected geometry, with very high sampling density (360 points in our case).

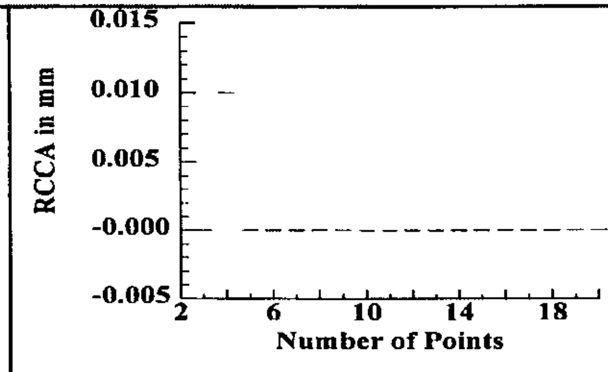
For circles, in order to replicate insofar as possible actual measurement procedure, for a given geometry (including form error), random noise level, sampling strategy, and number of points, approximately 50 experiments were conducted. Each experiment differed, in the case of uniform sampling, by a selection of a random angle commencing the measurement around a circumference. In the case of stratified sampling, the difference was further randomizing in that, within each segment of the circle, a random location was chosen.

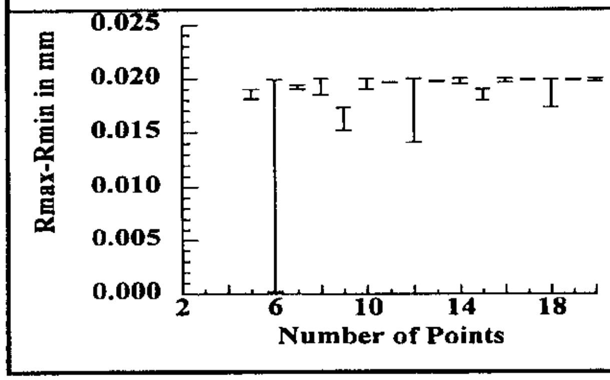
The results of the simulation was displayed in a series of graphs, like that shown in Figure 7., for a circle with three lobes, amplitude 10um measured on a machine with a random error three standard deviations, of 2.5 um, fit using the least squares algorithm and sampling using uniform sampling method. The vertical bars show the range of the values. The distributions of results were examined for several cases and found to be distinctly non-normal; the range therefore is the most meaning full parameter. The results from these experiments were used to develop sampling strategies for circles and to draw general conclusions. The results from uniform sampling were better than the results from stratified sampling. Results from one set of experiments using uniform sampling is analyzed in detail in the following section.

The results from least squares algorithm for a three lobed part with and without noise is given in Figure 7. The graphs clearly reveal a structure and the structure is preserved in the presence of noise. In order to get the correct radius at least 4 points must be sampled on a three lobed part. However, if the center of circle is also important then the number of points must be at least 5. In order to get the (R_{max} - R_{min}) close to the correct value the number of points must be a prime number (7, 11, 13, 17, or 19). If form error is present, selecting a prime number ensures the inclusion of extreme points irrespective of the starting angle. The higher the prime number, the smaller is the range of (R_{max} - R_{min}) value. The general conclusion that can be drawn based on the experiments for lobes 3 to 9 without noise is that the number of points must be at least n+1 where n is the number lobes to get the correct radius. The number of points must be at least n+2, where n is the number of lobes, to get the correct center. The correct out of roundness values got closer when the number of points is a prime number and the results


Perfect Geometry
Add Form Error
Number of Lobes
Lobe Amplitude
Sample Geometry
Uniform or Stratified Sampling
Number of Points
Random Starting Angle
Add Random Noise
Gaussian Distribution
3 σ Specified
Fit with Specific Algorithm
Repeat 40 Times
Compare Fit Parameters to True Values
Repeat with Different Number of Points
Display Results







Algorithm : Least Squares

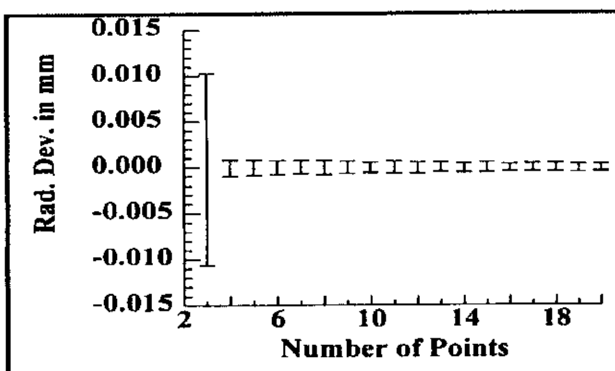
Radius : 10.00mm

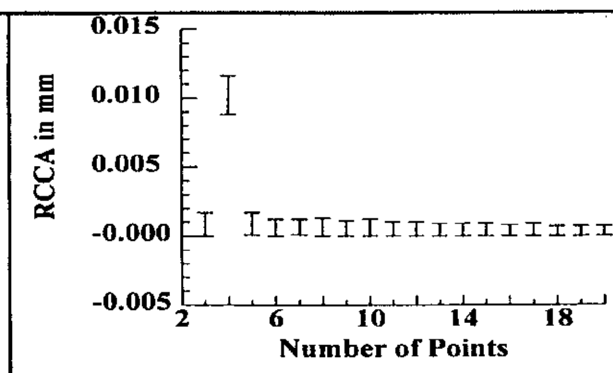
Random Noise : None

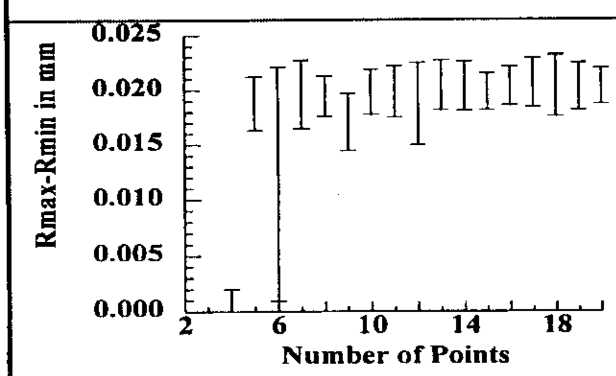
No. of Lobes/Lobe Amp : 3/10 microns

Sampling Scheme : Equi-Distant

Total Repetitive Runs : 40







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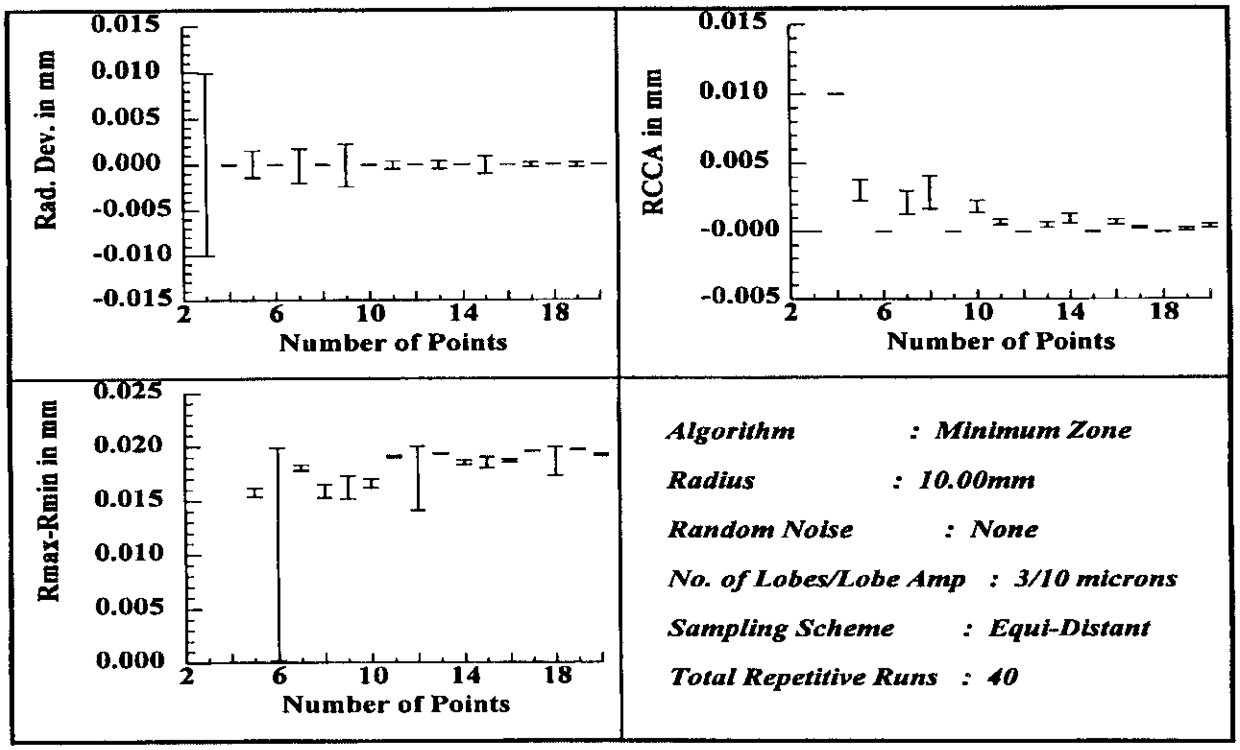
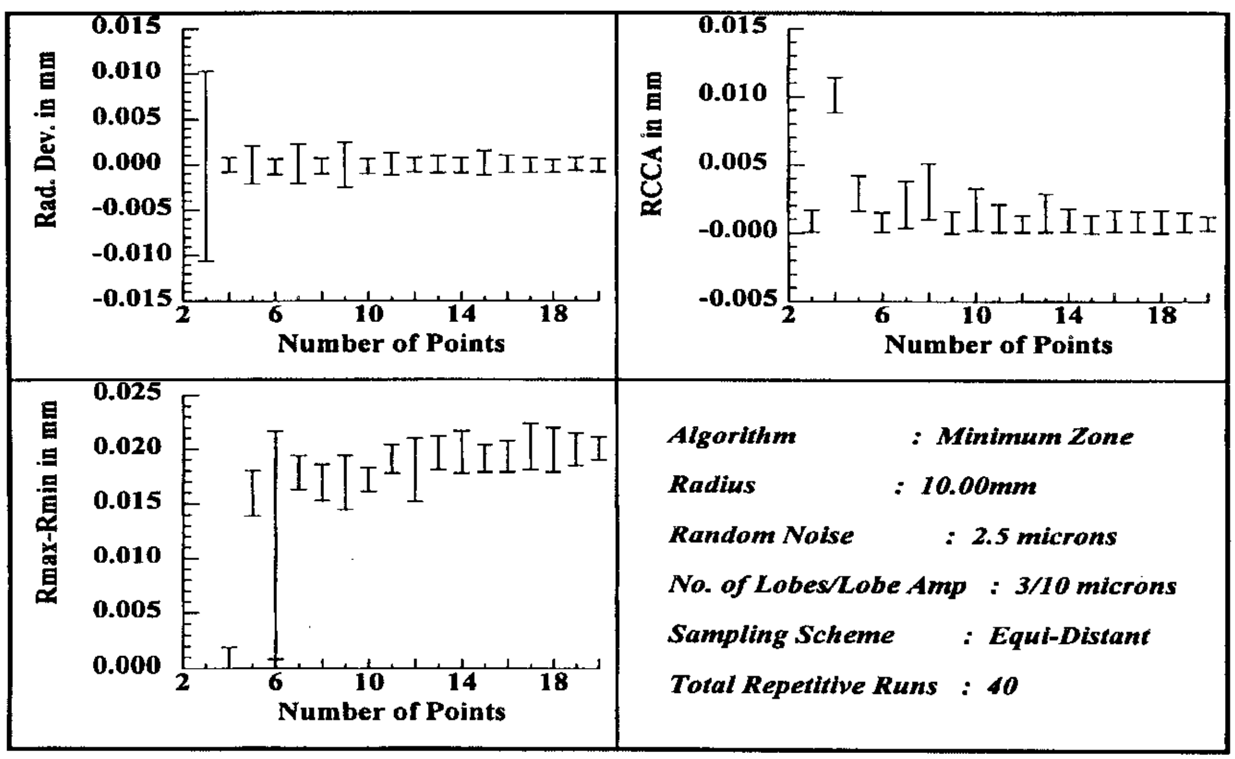

Rad. Dev. in mm
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
2
6
10
14
18
Number of Points
RCCA in mm
0.015
0.010
0.005
-0.000
-0.005
Rmax-Rmin in mm
0.025
0.020
0.015
0.010
0.005
0.000
Algorithm : Minimum Zone
Radius : 10.00mm
Random Noise : None
No. of Lobes/Lobe Amp : 3/10 microns
Sampling Scheme : Equi-Distant
Total Repetitive Runs : 40


Rad. Dev. in mm
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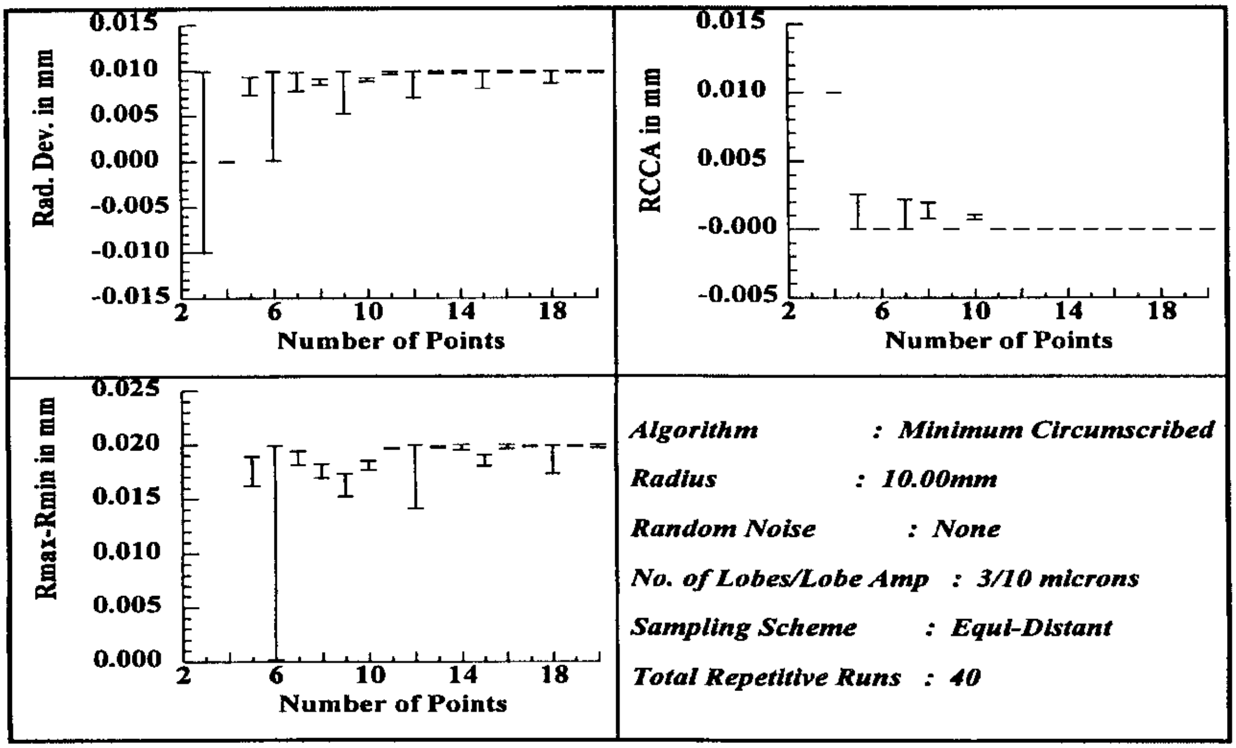
Rad. Dev. in mm
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-0.010
-0.015
2
6
10
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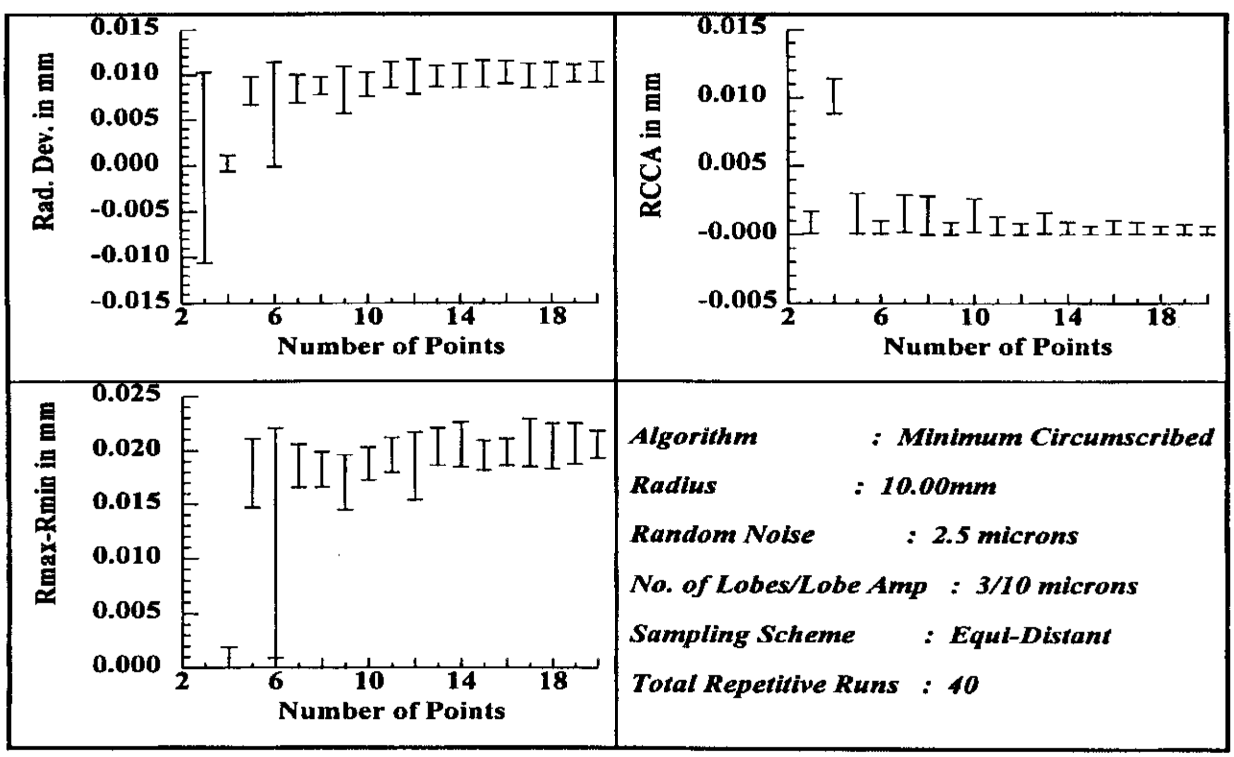
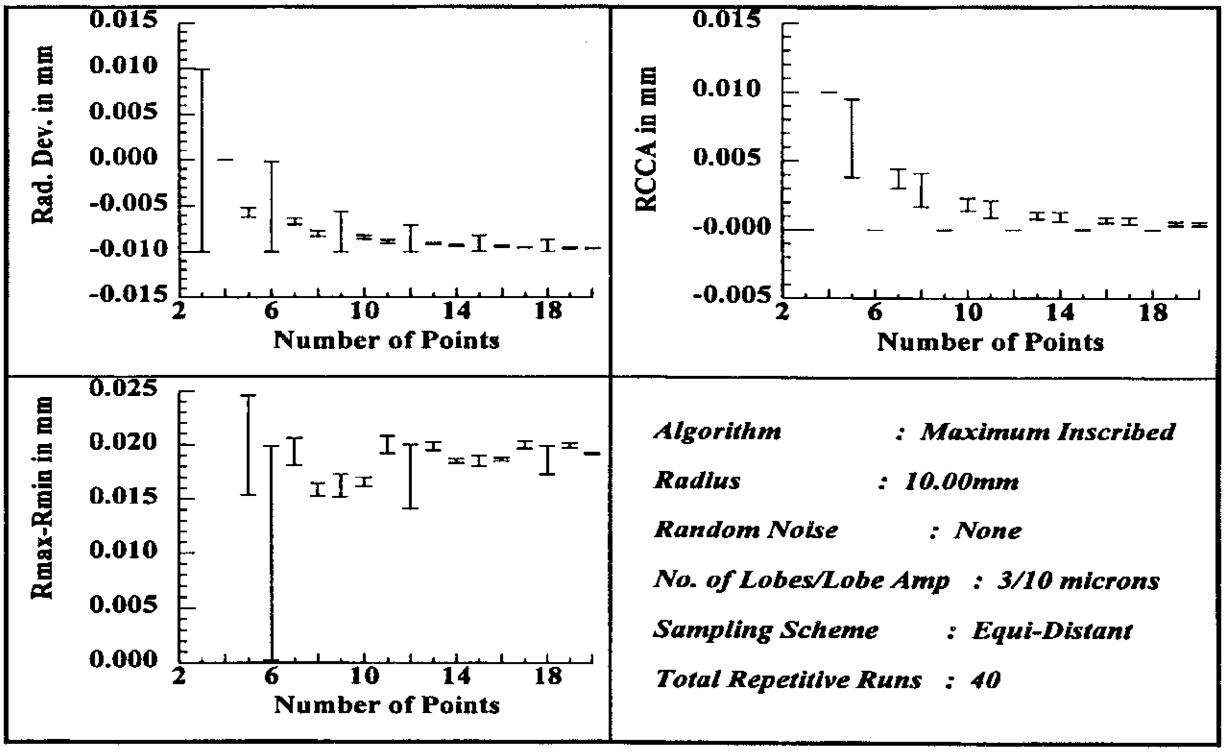
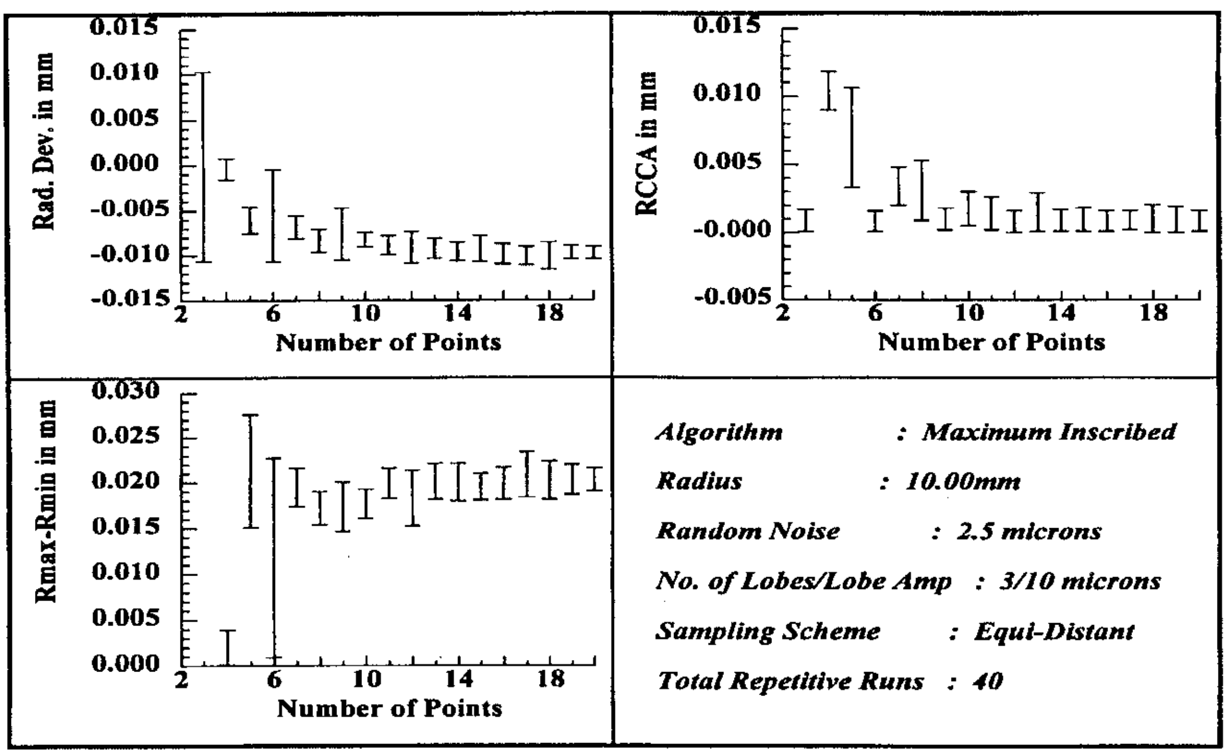
Number of Points
RCCA in mm
0.015
0.010
0.005
-0.000
-0.005
Rmax-Rmin in mm
0.025
0.020
0.015
0.010
0.005
0.000
Algorithm : Minimum Circumscribed
Radius : 10.00mm
Random Noise : None
No. of Lobes/Lobe Amp : 3/10 microns
Sampling Scheme : Equi-Distant
Total Repetitive Runs : 40


Rad. Dev. in mm
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
2
6
10
14
18
Number of Points
RCCA in mm
0.015
0.010
0.005
-0.000
-0.005
Rmax-Rmin in mm
0.025
0.020
0.015
0.010
0.005
0.000
Algorithm : Minimum Circumscribed
Radius : 10.00mm
Random Noise : 2.5 microns
No. of Lobes/Lobe Amp : 3/10 microns
Sampling Scheme : Equi-Distant
Total Repetitive Runs : 40


Rad. Dev. in mm
Number of Points
RCCA in mm
Number of Points
Rmax-Rmin in mm
Number of Points
Algorithm : Maximum Inscribed
Radius : 10.00mm
Random Noise : None
No. of Lobes/Lobe Amp : 3/10 microns
Sampling Scheme : Equi-Distant
Total Repetitive Runs : 40


Rad. Dev. in mm
Number of Points
RCCA in mm
Number of Points
Rmax-Rmin in mm
Number of Points
Algorithm : Maximum Inscribed
Radius : 10.00mm
Random Noise : 2.5 microns
No. of Lobes/Lobe Amp : 3/10 microns
Sampling Scheme : Equi-Distant
Total Repetitive Runs : 40

approached the correct values for larger number of points. All these requirement can be met if the number of points is a prime number that is greater than 2n. The factor two satisfies the Nyquist criteria and the prime number ensures the inclusion of extreme points. The actual choice of a prime number larger than the minimum is dictated by the precision of the machine, the lobe amplitude and the tolerance of the part. Figure 7b., shows the results from the experiment in which noise is included. The noise could be from the coordinate measuring machine or the part. The main conclusion from these results is that the structure seen in the results is the same as the one observed in the experiment without the noise. The noise introduces uncertainty in the results. Increased noise results in larger uncertainty. The sampling strategy for parts with systematic errors such as lobing is determined mainly based on the number of lobes. The noise levels were increased up to 6 times the lobe amplitude and the general conclusion drawn were still valid. This indicates that if form error is present the sampling strategy is mainly dictated by the form error even if excessive noise is present.

The results from minimum zone algorithm is shown in Figure 8. As in the case of least squares the results show a structure when form error is present and the presence of noise increases the uncertainty in the results. The correct results were obtained for radius and center when the number of points were 2n, where n is the number of lobes. The radius and the center positions have a large uncertainty when prime numbers are selected. The (R_{max} - R_{min}) values obtained using 2n points results in large uncertainty and this is due to the effect of starting angle while sampling the points on a circle. In general minimum zone algorithm is used for evaluating the form error, hence the number of points that give correct (R_{max} - R_{min}) is the ideal one. This indicates that the number of points must be a prime number to ensure the inclusion of the extreme points irrespective of the starting angle. If a prime number is selected then it must be greater than 4n to reduce the uncertainty in the radius and center values. Thus in the case of minimum zone the recommendation is to choose a prime number that is greater than 4n, where n is the number of lobes.

The results from minimum circumscribed algorithm and maximum inscribed algorithm are shown in Figures 9 & 10. In order to get correct results from these algorithms the sampled data must contain the extreme points and the starting point for the sampling must not influence the results. Selecting prime numbers satisfies this criteria. When the prime number is greater than 4n the radius and center position have a smaller uncertainty. Figure 11 shows the effect of starting angle when prime numbers are selected. The recommendation for maximum inscribed and minimum circumscribed algorithms is to choose a prime number that is greater than 4n, where n is the number of lobes.

The computer experiments clearly revealed that when form error is present there are some preferred number of points. Irrespective of the algorithm used, if the number points selected is a prime number and is greater than 4n (2n for least squares), where n is the number of lobes, the correct results will be obtained. The presence of noise increases the uncertainty and the uncertainty value increases for increased value of noise.

5.0 Validation of Computer Simulation

An extensive amount of experimentation had to be done with parts having different characteristic errors and under different conditions of the influential parameters. Hence, computer simulation of the measurement system was essential for our problem. This gave the flexibility of isolating the errors in the part to study them thoroughly. In order to check the simulated experiments, a large number of parts were measured on our CMM and the results compared to those obtained from the computer experiments. All together, 20 features on 16 different parts were measured with the CMM. These parts were made from common materials, such as aluminum, steel and brass, and were believed to be good representation of the possible form errors for manufacturing processes. Although all of the parts probably had form error, it was found that cylindrically-ground, centerless ground, honed and some turned or bored parts had errors that were as small as, or smaller than, the machine probing errors on our machine and therefore could not be detected. On those parts where form error could be detected, namely drilled, turned, bored, circularly-contoured, and some reamed parts, the most prominent lobes were two, three, four and six.

In addition to this qualitative comparison, quantitative comparison between the CMM results and the simulated parts was done. One of the best examples was that of a three-lobed part, where the measurements almost precisely matched our simulated results. In Figure 11. the graphical results for location, diameter and form error of the simulated part and the real part are compared. As can be seen, the behavior of the computer simulation is quantitatively and qualitatively nearly identical to that of the actual measurement which validates the computer simulation procedure.

6.0 Conclusion:

The computer simulation and validation of sampling strategies for circles showed rather conclusively, that most stable results were obtained using equi-angular distribution of points with the number of points being considerably more than the number of lobes. The main conclusion is that form error (lobing), if present dictates the sampling strategy. When the number of points is a prime number greater than 4n (2n in the case of least squares), where n is the number of lobes, correct results are obtained. The effect of random error whether from the part or the measuring machine were also quite predictable using computer simulation. Random error simply added uncertainty in the particular fit parameter, proportional to the size of the random error added to the form error.

7.0 References:

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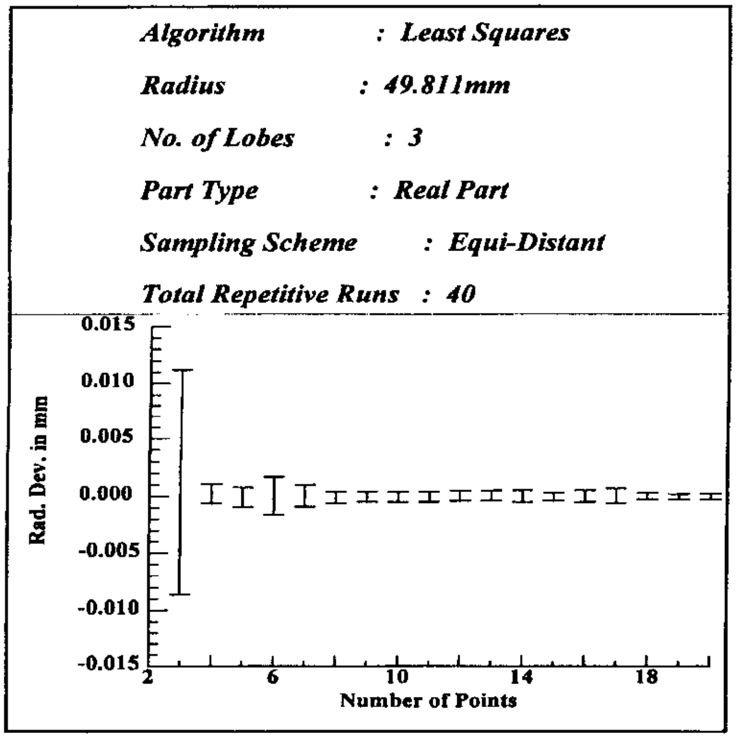
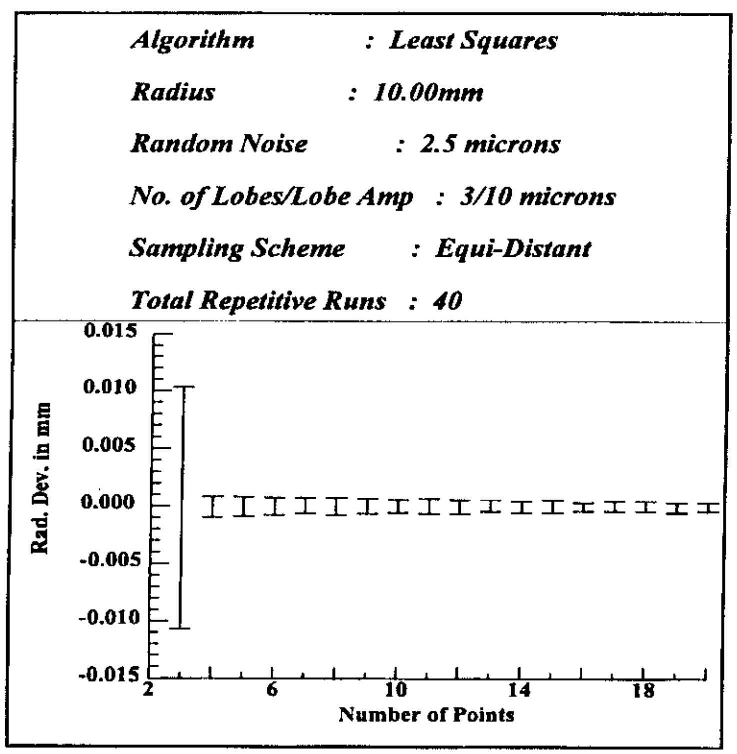

Algorithm : Least Squares
Radius : 49.811mm
No. of Lobes : 3
Part Type : Real Part
Sampling Scheme : Equi-Distant
Total Repetitive Runs : 40
Rad. Dev. in mm
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
2
6
10
14
18
Number of Points


Algorithm : Least Squares
Radius : 10.00mm
Random Noise : 2.5 microns
No. of Lobes/Lobe Amp : 3/10 microns
Sampling Scheme : Equi-Distant
Total Repetitive Runs : 40
Rad. Dev. in mm
0.015
0.010
0.005
0.000
-0.005
-0.010
-0.015
2
6
10
14
18
Number of Points

Algorithm **: *Least Squares***

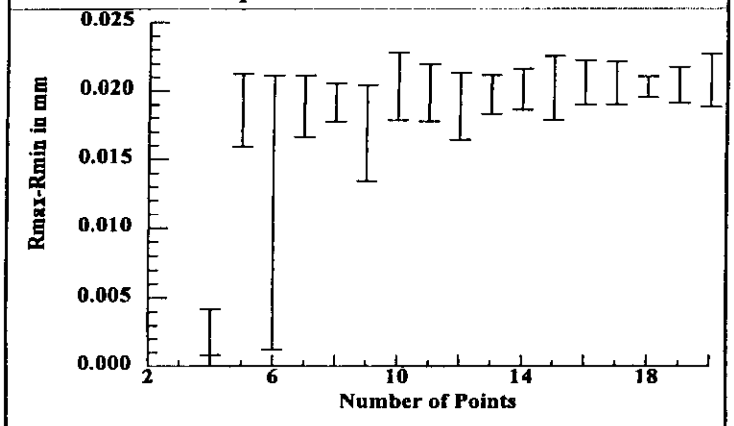
Radius **: *49.811mm***

No. of Lobes **: *3***

Part Type **: *Real Part***

Sampling Scheme **: *Equi-Distant***

Total Repetitive Runs **: *40***



Algorithm **: *Least Squares***

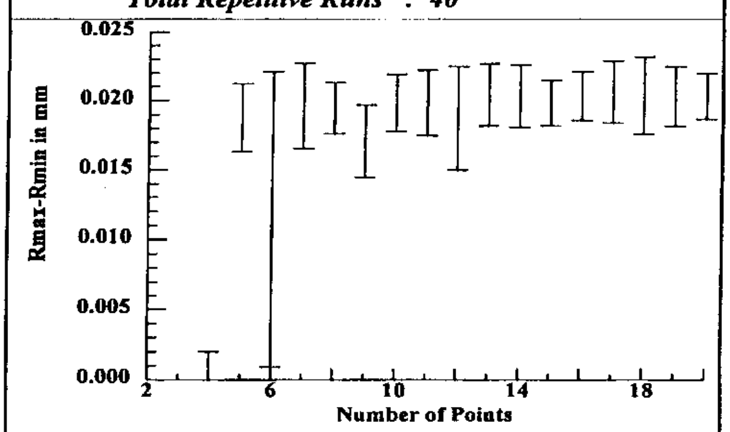
Radius **: *10.00mm***

Random Noise **: *2.5 microns***

No. of Lobes/Lobe Amp **: *3/10 microns***

Sampling Scheme **: *Equi-Distant***

Total Repetitive Runs **: *40***



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