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A note on the three-point method for roundness measurement

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Abstract

This technical note presents two enhancements to three-point method for roundness measurement. They enable measuring a larger bandwidth and also present a logical progression from two-point profile to three-point roundness measurement using the combined method. Simulated profiles with and without step variations are used to demonstrate these improvements.

Keywords: DFFT reconstruction; Harmonics; Simulated profile

1. Introduction

Three-point method for roundness detection [1] is commonly used to separate the influence of spindle error from part error. This technique has a major limitation in that certain harmonics are not recoverable from the combined signal. There are also other issues with this approach. Reconstruction of the profile from the combined signal is subject to uncertainties. Accurate estimation requires that the part be sampled at exactly the probe spacing. But this prevents surface wavelengths smaller than probe spacing to be recorded. DFFT method is useful in recreating the signal but introduces distortion when there are sharp features.

The combined three-point method (C3P) has been proposed [2] to overcome the issue of distortion in presence of sharp features. This method requires a reference signal and uses the DFFT reconstruction, which itself is subject to harmonic suppression. Also, the C3P method uses a second order polynomial interpolation during reconstruction, which introduces distortions in certain cases. In this context, we propose two enhancements to three-point roundness measurement. First we present an approach to enhance the measurable bandwidth using the DFFT method. Then, we modify the inclination method [2] using limacon fitting to ensure better performance. These result in greater scope for the C3P method. Simulated profiles with and without steps are used to illustrate the improvements. The next section outlines a brief summary of two-point method for profile measurement. This is followed by a discussion on improvements to three-point method, results and conclusions.

2. Two-point profile measurement

The objective of the two-point method [3] is to remove the influence of z directional error when scanning a profile in x (see Fig. 1). If a surface is described by function $f(x)$, D is the probe interval and S is the sampling period, output of probe A is m_A and probe B is m_B, then

$$m_A(x_n) = f(x_n) + e_z(x_n) \quad (1)$$

$$m_B(x_n) = f(x_n - D) + e_z(x_n) \quad (2)$$

The difference function

$$m(x_n) = m_A(x_n) - m_B(x_n) \quad (3)$$

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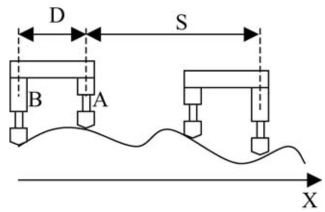

Fig. 1. Two-point profile measurement.

cancels the error e_z. An approximate derivative can be defined as:

$$m'(x_n) = \frac{(m_A(x_n) - m_B(x_n))}{D} \quad (4)$$

The original profile Z is recreated by integrating Eq. (4) using the iterative equation given in Eq. (5):

$$Z(x_n) = \sum_{i=1}^{n} m'(x_i)S = Z(x_{n-1}) + m'(x_{n-1})S \quad (5)$$

Because $Z(x_0)$ is unknown, its value is assumed to be zero. The resulting Z profile is therefore inclined to the X-axis and the slope is removed by least-squares fitting to obtain the Z profile. Previous reported work [3] has referred to this method as the inclination method when $S = D$ and generalized method when $S \neq D$. When $S = D$, the profile is reconstructed accurately because the differential is no longer approximate. The problem with this case is that surface wavelengths smaller than the probe spacing D cannot be captured.

3. Combined three-point method for roundness

The objective of the three-point method [2] for roundness is to remove errors associated with the spindle during profile measurement. In this case, let $r(\theta)$ represent the surface, m_A, m_B and m_C represent the probe outputs, ϕ represent the angle between the probes and e_x and e_y represent the X and Y components of the spindle error (see Fig. 2). Then,

$$m_A(\theta_n) = r(\theta_n) + e_x(\theta_n) \quad (6)$$

$$m_B(\theta_n) = r(\theta_n - \phi) + e_x(\theta_n)\cos(\phi) + e_y(\theta_n)\sin(\phi) \quad (7)$$

$$m_C(\theta_n) = r(\theta_n - 2\phi) + e_x(\theta_n)\cos(2\phi) + e_y(\theta_n)\sin(2\phi) \quad (8)$$

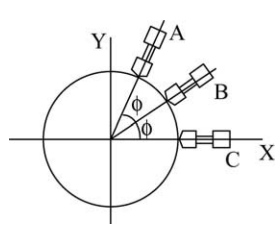

Fig. 2. Three-point roundness measurement.

Then, as before in the profile method, a differential output

$$\begin{aligned} m(\theta) &= m_A - 2\cos(\phi)m_B + m_C \\ &= r(\theta) - 2\cos(\phi)r(\theta - \phi) + r(\theta - 2\phi) \end{aligned} \quad (9)$$

is defined to remove the errors. The problem is to obtain the profile $r(\theta)$ from differential output given in Eq. (9).

3.1. DFFT method

A solution to this problem [1] is to take the FFT of both sides of Eq. (9) to obtain

$$M = R(1 - 2\cos(\varphi)\,\mathrm{e}^{-jw} + \mathrm{e}^{-2jw}) = RH \quad (10)$$

The profile can then be obtained by taking the inverse transform of (M/H). This method, although known, is often not suitable because the transfer function H drops to zero at certain harmonics [1]. Therefore, the profile cannot be reconstructed accurately if these harmonics are present in the surface. To overcome this problem, we study the choice of probe angles as a potential solution.

While the transfer function H drops to zero at certain harmonics for any probe angle in the continuous domain, the same is not true in the discrete domain. Because FFT captures only certain discrete frequencies, there are only selected probe angles for which the transfer function H completely suppresses certain harmonics. For purpose of simplicity, only integer probe angles are considered here. For typical applications where harmonics from 1 to 100 are relevant, probe angles (only from 1° to 120° are listed) belonging to the following set result in complete suppression of certain harmonics:

Probe angle (°): {4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 28, 30, 32, 35, 36, 40, 42, 44, 45, 48, 50, 52, 54, 55, 56, 60, 63, 64, 65, 66, 68, 70, 72, 75, 76, 78, 80, 81, 84, 85, 88, 90, 92, 95, 96, 99, 100, 102, 104, 105, 108, 110, 112, 114, 115, 116, 117, 120}

Thus, any *integer* probe angle not belonging to the set above result in complete transmission of the signal by pushing the zeros of the transfer function well beyond the 100th harmonic as shown in an example in Fig. 3. However, it is also pointed out that when |H| is almost but not identically zero, those harmonics will be sensitive to uncertainties in probe angle value. This is illustrated in Fig. 4. A surface with a 45th harmonic is reconstructed using three probes assumed to be separated by 23° each, while the true probe angle is 23.25° (to simulate probe angle uncertainty). Also shown is a profile with a 44th harmonic that is reconstructed using a probe angle of 23°, while the true probe spacing is 23.25°. It is seen that the surface with 45th harmonic is much more sensitive to probe angle uncertainties because |H| for that harmonic is much smaller.

There is another problem with the use of the DFFT method, as reported by Gao and Kiyono [2]. This method

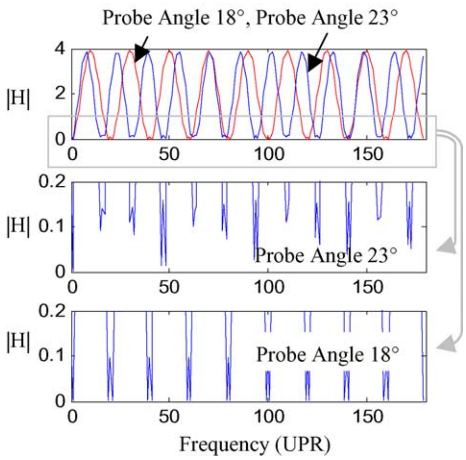

Fig. 3. Effect of probe angle on zeros of transfer function H.

does not perform well in presence of spikes or sharp features in the surface. To overcome this problem, they propose a combined method that utilizes the low frequency components from the DFFT method as a reference. A modification to this method is explained next.

3.2. The inclination method

When the sampling interval is equal to the probe spacing, reconstruction can be achieved by integration. That is:

$$z(\theta) = m(\theta) + 2\cos(\phi)r(\theta - \phi) - r(\theta - 2\phi) \quad (11)$$

The first two points in Z are set to zero and the profile is reconstructed. The key point to note is that while such reconstruction resulted in an inclined profile in the two-point method, reconstruction in the three-point method results in an eccentric circle. Therefore, we remove a limacon

$$a\cos(\theta) + b\sin(\theta) - r \quad (12)$$

using the least-squares method to obtain the true profile Z. Previous reported literature [2] suggests a second order fit to obtain the original Z profile. Fig. 5 shows an example profile

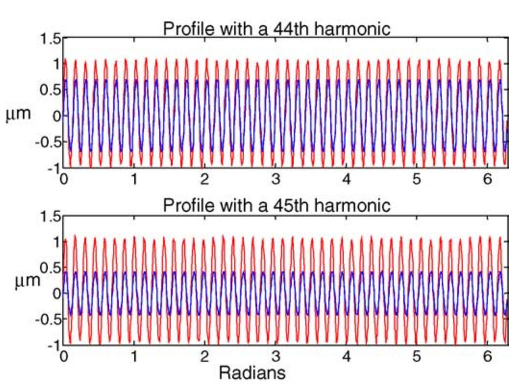

Fig. 4. Effect of probe angle uncertainty on reconstructed profile. Original profile has amplitude of 1. Reconstructed profile has smaller amplitude (0.7 µm for 44th harmonic and 0.4 µm for 45th harmonic).

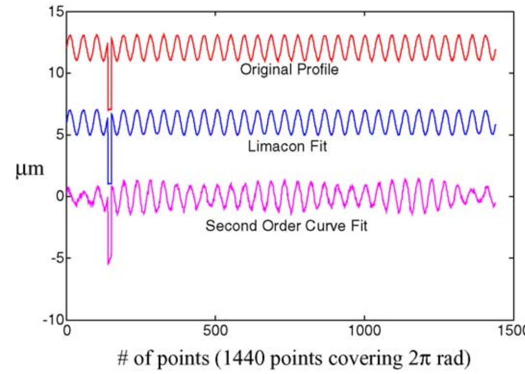

Fig. 5. Effect of curve fitting criteria on reconstructed profile.

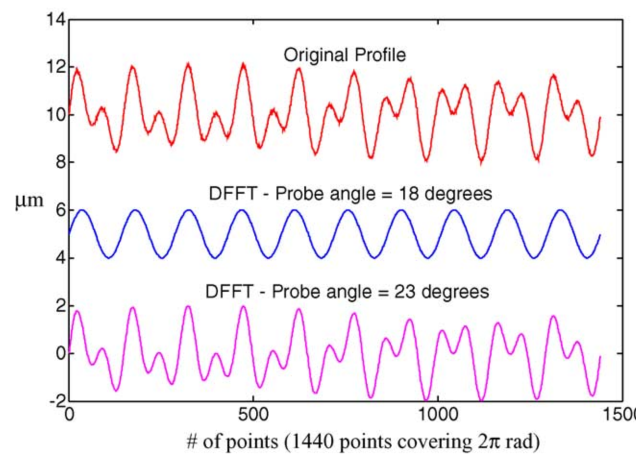

Fig. 6. DFFT output for probe angles 18° and 23°.

along with a limacon fit and a second order fit to illustrate the case.

The major issue with this method, as with the two-point method is that surface wavelengths smaller than the probe interval cannot be captured. To overcome this method, the combined method was proposed earlier [2].

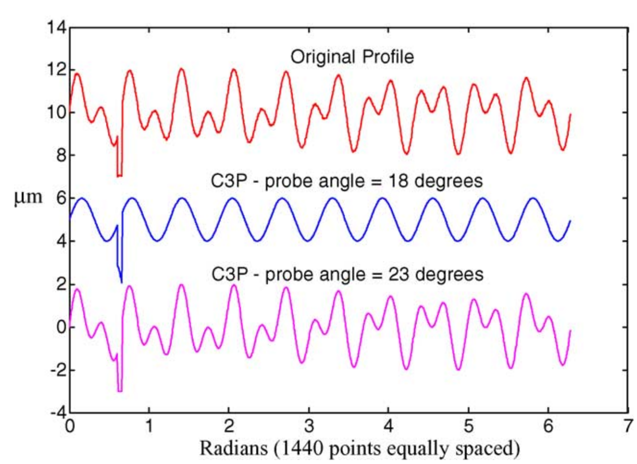

Fig. 7. C3P output for probe angles 18° and 23°.

3.3. The combined method

The combined method essentially samples at a smaller interval than the probe spacing. Then, the resulting data is partitioned into groups, with each group having points that are at exactly the probe spacing. Therefore, each group can be reconstructed using the inclination method. The problem then is to align the groups with each other. This is achieved by using the profile from the DFFT method as a reference. Because this method uses the DFFT as a reference, it is susceptible to failure when the part harmonic falls in the dead zone. The algorithm of the C3P can be found in [2], but to enhance the performance of this method, we recommend that:

1. Careful choice of probe spacing based on the set outlined earlier,
2. Eccentric circle fitting be used for each group as opposed to second order curve fitting.

4. Results

A simulated profile with two sinusoids (10th and 19th harmonic) is shown in Fig. 6. Probe spacing of 18° and 23° is considered. The reconstructed profile by DFFT method fails to detect the 19th harmonic when probe angle is 18°. The reconstruction is perfect for 23° probe angle. The same profile with a step feature is shown in Fig. 7. C3P reconstruction is perfect for 23° as opposed to 18°.

5. Conclusions

This note reports on two developments in three-point method that enhance the performance of the combined method. Utilizing carefully chosen angles for probe spacing based on the set outlined earlier increases the measurable bandwidth, thus allowing greater scope for the DFFT method. Also, an improvement to the inclination method is presented that recognizes the fact that iterative reconstruction produces eccentricity in the output that can be corrected using limacon fitting. These enhancements together help increase the scope of the C3P method.

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