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REVIEW ARTICLE

A Comprehensive Review on Computational Techniques for Form Error Evaluation

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Abstract
In industries, precise measurement of different form errors is critical and challenging for stringent geometric and dimensional control in manufactured part. These form errors have significant influence on the functional performance of an industrial product that arises due the inherent invariability in measurement techniques and manufacturing devices. Several research have been conducted for evaluation of important form errors such as straightness, flatness, circularity, cylindricity and sphericity using different computational methods. The need of computational methods is justified as they minimize calculation time, human errors and provides improved tolerance values in assessment of form errors. In the same context, this paper presents a comprehensive review and discussion on different computational techniques for distinctive form error evaluation in engineering components. The present work mainly focused on aspects of mathematical formulations and the computational techniques i.e., traditional methods and advanced optimization algorithms, employed for precised evaluation of these errors. Based on the detailed review, several future research directions were described. Finally, last section presents concluding remarks on computational methods in modelling and precise evaluation of form error in manufactured components.

Keywords Form error evaluation · Computational techniques · Metaheuristic algorithm · Measurement

1 Introduction

In metrological context, the manufacturing of precised features and dimensions is ever increasing in modern competitive environment and have gained significant interest among manufacturing industries to conform the rigorous dimensional and form tolerance requirements [1]. The features created in a component through any manufacturing process may vary in systematic or random means depending on the inherent variability in machines [2]. Therefore, maintaining the quality and interchangeability among manufactured components and features as per the design specifications and tolerances becomes equally important for functioning individually or in an assembly [3]. In early days, the form and dimensional errors are measured and evaluated utilizing the traditional gauges, instruments, and calipers [4]. In recent years, the measurement and inspection of manufactured parts have been governed by coordinate measuring machines (CMMs) that plays a significant role in automatizing and reducing uncertainty in quality control process [5]. The CMM measured data realized for different features are in the form of x, y and z coordinate values, with increased accuracy owing to its capability in realizing large datasets of the sampled surface [6]. The datasets are further processed and evaluated surface metrological feature based on complex verification algorithms to verify their conformance as per specified tolerance. The traditional steps for form error evaluation is shown in Fig. 1.

The verification algorithms generally follow least square method (LSM) to evaluate different form errors of manufactured surface features because of its ease in computation and distinctiveness in solution [7, 8]. Although, LSM is widely used for determining geometric and dimensional tolerances, however, it does not strictly adhere to the minimum zone solution required as per the ANSI Y14.5 standard [9, 10]. The LSM techniques is easy to code however lead to overestimation of tolerances which may results in rejection of potentially functional parts, thus resulting in economic loss [11]. The ISO standard while stating minimum zone attribute does not specify any particular technique for determining the form errors [12]. For attaining exact minimum zone

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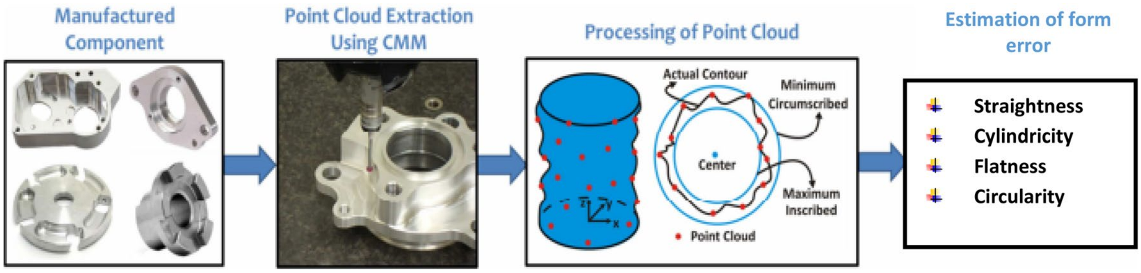

Fig. 1 Schematic showing traditional steps during form error evaluation [15]

error, several methods have been developed in past studies involving specific analytical and computational techniques. The analytical methods make use of mathematical characteristics of underlying objective function for optimization. In contrast, the computational techniques exploit numerical value of specific objective function for optimization. Though the analytical methods are useful, yet the computational techniques are advantageous when analytic properties of the objective function are difficult to realize [13]. In addition, the computational techniques may provide rapid results when the complexity or non-linearity of the minimum zone method need to be solved efficiently in form error evaluation [14]. The computational techniques are further classified as: traditional algorithms and non-traditional (advanced) algorithms for improving the accuracy in form error evaluation.

The traditional algorithms used in literature for solving complex and non-linear form error problems are downhill simplex search, linear programming, iterative reweighted least square algorithm, discrete and linear Chebyshev approximation and finite-differences derivative descent approach [16]. These algorithms are ubiquitous and computationally efficient; however, they does not provide accurate results owing to their mathematical approximations. These algorithm also suffers to attain global best solution in case of higher nonlinearity where a number of local solutions are also present [17]. To resolve these issues, advanced optimization algorithms are utilized which are flexible, adaptive, gradient free, can find optimal solutions and thus successively employed by researchers in the field of metrology. In the last decade, many optimization techniques such as genetic algorithm (GA), particle swarm optimization (PSO), artificial bee colony (ABC), Differential evolution (DE), Beetle search algorithm, ant colony algorithm and many improved optimization algorithms had been used for determination of form error. These advanced optimization algorithms have their own advantages and disadvantages. GA effectively can find the solution of complex real life objective problems with different chromosome encoding, however, it requires tuning of various operators such as crossover, mutation, selection etc. [18]. The PSO algorithm is swarm-based algorithm inspired by swarm movement in search space also have to tune some parameters and may fall in local optima some time [19]. The ABC algorithm based on foraging behavior of honeybee can obtain high quality solutions, but it suffers from slow convergence speed [20]. Though, the DE is contrast to GA in terms of mutation process and selection, however, it also have parameters to be defined at the beginning [21].

Aforementioned computational techniques i.e., traditional, and advanced algorithms are established algorithms and successfully applied in the last two decades for scientific research, engineering, and medical applications [22, 23]. Moreover, these computational algorithms are also effectively utilized in determining qualitative characteristics of surface metrological features including straightness, flatness, circularity, cylindricity and sphericity. These computational techniques are proving to be milestone in form error evaluation and still no work is available that may educate researchers about the qualities and performance of these techniques for solving such complex problems. In the same context, this review paper makes an attempt to identify and cover all aspects of computational techniques available in literature that may have been utilized in assessment of any form errors for different manufactured components. Furthermore, some hybrid techniques combining the advantage of two techniques, have also been studied and discussed in form error evaluation. The next section introduces the basic terminology and mathematical formulation of different form errors in literature. Further, in subsequent section each form error i.e., straightness, flatness, circularity, cylindricity and sphericity, is discussed and summarizes for application of different computational techniques involving traditional and advanced optimization algorithms. Later, application areas for various form error evaluation is discussed. Finally, future research directions are discussed and summarized based on the comprehensive review. Figure 2 depicts the publication trend in form error evaluation using different computational techniques.

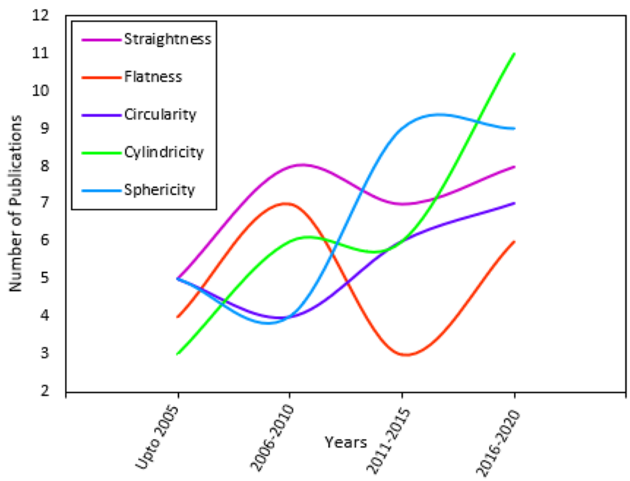

Fig. 2 Publication trend showing publications on computational techniques in form error evaluation over the years

2 Mathematical Formulations

This section presents the form error definition and mathematical formulation for straightness, flatness, circularity, cylindricity, sphericity and conicity available in literature.

2.1 Straightness Error Formulation

In common terms, the straightness error can be defined as the deviation from the reference imaginary perfect straight line. As per ISO, the minimum zone straightness error is referred to as the least distance between two ideal parallel lines enclosing the actual straight line created from the measured points as shown in Fig. 3 [24]. The measured line element of any surface is represented by n data points, where the x, y and z coordinates of i th data point $(i = 1, 2, \ldots n)$ are given by either (x_i, y_i) or (x_i, y_i, z_i) in two and three dimension, respectively. Let the ideal straight line in 2-dimensions is represented as: $y = l_m x + y_0$, where l_m and y_0 are denoted as defining factors of the equation. The vertical deviation measured along y axis may be considered as: $V_d = y_i - (l_m x_i + y_0)$. Although, the vertical deviation can be easily determined using LSM, however, the actual straightness error is measured by normal deviation only [25, 26]. The normal deviation N_d equation of the corresponding point from reference edge can be formulated as:

$$N_d = \frac{y_i - (l_m x_i + y_0)}{\sqrt{1 + l_m^2}} \tag{1}$$

If the maximum and minimum value among these deviations are denoted as $N_{d\,\max}$ and $N_{d\,\min}$, then the straightness error can be determined as: $St_e = N_{d\,\max} - N_{d\,\min}$.

2.2 Flatness Error Formulation

The flatness can be considered as an extension of straightness error in three dimensions. In practice, flatness error may be defined as the deviation of measured planar surface from the reference imaginary perfect planar surface. In terms of minimum zone method, the flatness error may be defined as the tolerance zone between two parallel planes within which the measured plane points must lie as depicted in Fig. 4 [27]. The i th data point from any measured planar surface is represented by (x_i, y_i, z_i). As per the ISO standard, all measured data points for $i = 1, 2, \ldots n$, must lie in between two ideal planes which are represented as

$$z = a_0 x + b_0 y + c \tag{2}$$

where a_0, b_0 and c are designated as defining parameters. The flatness determined in terms of vertical deviation

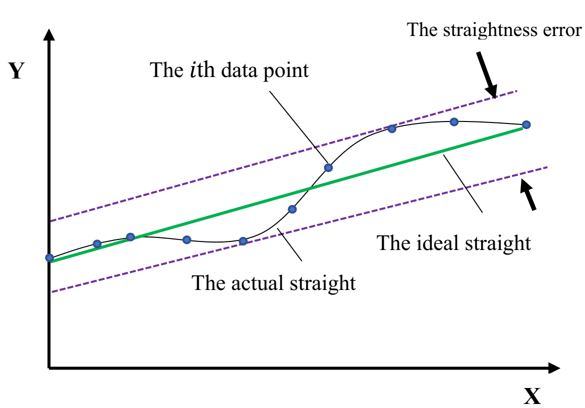

Fig. 3 Schematic for Straighetss error

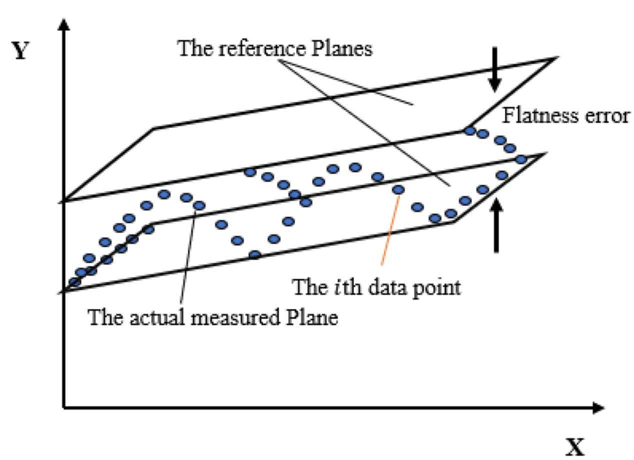

Fig. 4 Schematic for flatness error evaluation

for any measured point from given surface is given as: $V_d = z_i - \left(a_0 x_i + b_0 y_i + c\right)$. The normal deviation of measured point $\left(x_i, y_i, z_i\right)$, from the reference plane can be calculated as:

$$N_d = \frac{z_i - \left(a_0 x_i + b_0 y_i + c\right)}{\sqrt{1 + a_0^2 + b_0^2}} \tag{3}$$

The maximum and minimum deviations of all measured points from reference planes are denoted as $N_{d\,\max}$ and $N_{d\,\min}$, then the flatness error tolerance zone can be determined as: $F_e = N_{d\,\max} - N_{d\,\min}$.

2.3 Circularity Error Formulation

The circularity error formulation and evaluation is important in context of mechanical parts, as circular feature forms most basic geometric element of any component [28]. As per ISO standards, circularity error referred to as the minimum radial deviation between two concentric circles enclosing all the measured points of actual circular feature as shown in Fig. 5 [29]. The radial deviation is computed along the radius from the circle's center to the measured point of actual profile. In addition, there will be several sets of concentric circles that can enclosed the measured profile data, however only one combination will have the minimum radial deviation. Assuming the ideal center of minimum zone circle is at $\left(a_1, b_1\right)$, the radial distance from measured profile i th point $\left(x_i, y_i\right)$ and to the center is given as:

$$R_i = \left(\left(x_i - a_1\right)^2 + \left(y_i - b_1\right)^2\right)^{\frac{1}{2}} \tag{4}$$

where a_1, b_1 are defining parameters. If $R_{\max}$ and $R_{\min}$ are the maximum and minimum radius of circle having all measured profile data points, respectively, then the circularity error can be mathematically defined as:

$$C_e = f\left(a_i, b_i\right) = R_{\max} - R_{\min}. \tag{5}$$

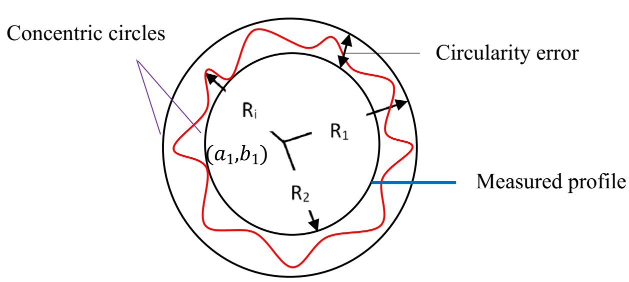

Fig. 5 Schematic for circularity error evaluation

2.4 Cylindricity Error Formulation

The cylindricity error evaluation has a significant impact on the functioning of assemblies, rotation and wear resistance. Based on minimum zone criterion, cylindricity error is defined as the radial difference between two concentric cylinders enclosing all extracted cylindrical surface data points as shown in Fig. 6 [30]. Thus, an ideal cylinder is utilized having the axis of this ideal cylinder may be defined by an arbitrary point $\left(x_0, y_0, z_0\right)$ with direction vector as (l, m, n). The ideal axis may be defined mathematically as:

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \tag{6}$$

where $x_0, y_0, z_0, l, mandn$ are designated as defining parameters of the equation. If the i th point of measured surface are denoted as $\left(x_i, y_i, z_i\right)$, then the radial distance from the center of minimum zone cylinder and point (x_i, y_i, z_i) on measured cylinder profile can be evaluated as:

$$C_d = \sqrt{\frac{a^2 + b^2 + c^2}{l^2 + m^2 + n^2}} \tag{7}$$

where $a = \left(y_i - y_0\right) \cdot n - \left(z_i - z_0\right) \cdot m$; $b = \left(z_i - z_0\right) \cdot l - \left(x_i - x_0\right) \cdot n$; $c = \left(x_i - x_0\right) \cdot m - \left(y_i - y_0\right) \cdot l$.

Then, if $C_{d\,\max}$ and $C_{d\,\min}$ are the maximum and minimum radial distance of cylinder having all measured

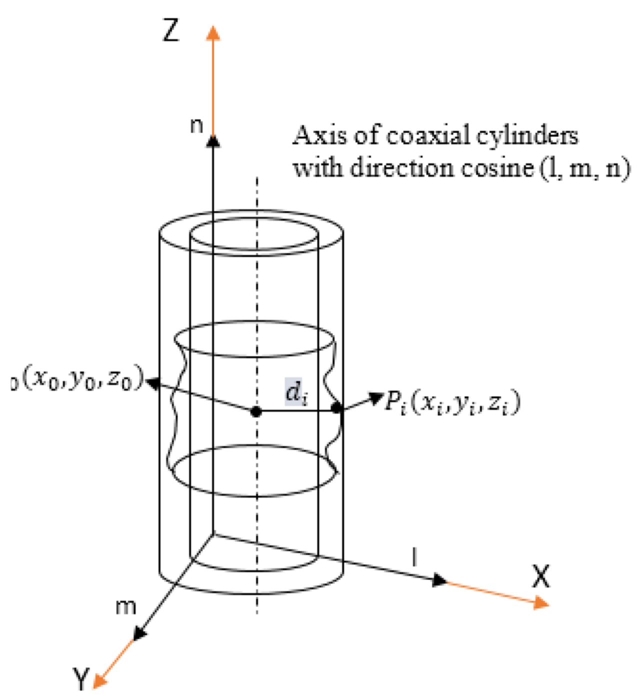

Fig. 6 Schematic for cylindricity error

profile data points, respectively then the cylindricity error can be evaluated as $Cy_e = C_{d\max} - C_{d\min}$.

2.5 Sphericity Error Formulation

Earlier, the above four form errors are mostly determined in industries, however, the evaluation of sphericity error is also need of hour due to immense utilization of spherical features especially in bearing industries. The sphericity is an important form error that may affect the rotating and assembly parts significantly and may hamper intended functionalities of mechanical parts [31]. Though, the concept of sphericity error is not included in the basic form errors and is not followed by ISO standards [29], the definition based on minimum zone solution can be formulated as the radial distance between two concentric spheres enclosing all the data points of measured sphere profile as shown in Fig. 7. Suppose the ideal center of minimum zone sphere is at $\left(x_0, y_0, z_0\right)$, the radial distance from measured profile i th point (x_i, y_i, z_i) and to the center is given as:

$$R_i = \left(\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 + \left(z_i - z_0\right)^2\right)^{\frac{1}{2}} \tag{8}$$

where x_0, y_0, z_0 are defining parameters. If $R_{\max}$ and $R_{\min}$ are the maximum and minimum radius of sphere having all measured profile data points, respectively, then the sphericity error can be mathematically formulated as:

$$S_e = R_{\max} - R_{\min}. \tag{9}$$

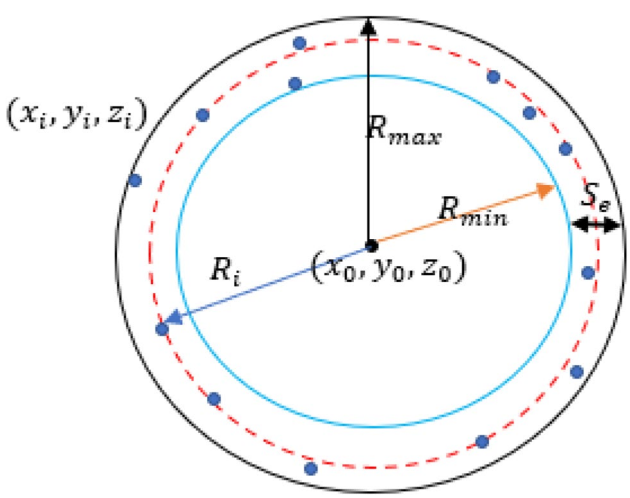

Fig. 7 Schematic for sphericity error evaluation

3 Computational Techniques for Various Form Error Evaluation

From the above section, it is evident that the minimum zone method evaluate accurately different form errors in comparison to least square method. However, the utilization of minimum zone method brings non-linearity in the form error equation, which requires different computational techniques for their solution. In this section, an attempt has been made to present the discussion on literature available for all form error evaluation using traditional and advanced optimization algorithms.

3.1 Straightness Error Evaluation Using Computational Techniques

3.1.1 Traditional Techniques

For obtaining the minimum zone solution, several research works are available that utilizes traditional computational techniques in the form error metrology. In the same context, computational geometry-based technique was proposed in [32] for fast and effective processing of the form data along with CMM data and further evaluation of straightness error. Initially, the data points are divided into two convex hull sub-sets and then antipodal set of data points are utilized for finding the minimum zone straightness error. The results after comparing with literature reveals that the proposed computational geometry-based technique is computationally less complex, require short time for execution and has robust performance providing good solution. The computational method based on convex hull set and judgement formula was also presented in [33, 34] for evaluating spatial straightness error, that proves computationally efficient and takes less time. In [35], the authors applied region-elimination search algorithm for straightness error evaluation dividing the line element data points into four different but similar regions. The elimination of data points is performed with iterations in the range of Δ, Δ/2 and Δ/4 based on optimal error value in the specified tolerance zone. When compared with past studies, the region-elimination search algorithm effectively minimized the sample points and provided desired level of accuracy in straightness error evaluation. In [36], a successive quadratic programming (SQP) techniques was applied for accurate and fast evaluation of non-linear spatial straightness error. The results of SQP technique are well within the range of 10–3 mm and are in accordance with literature.

A fast geometrically based computational algorithm is proposed in [37] for straightness error evaluation in which the data points are processed by creating symmetric beta form error that is added to a line having randomly distributed slope and intercept. The results show that the proposed computational algorithm outperformed well-known sorting algorithms in terms of computational complexity. In [38], the authors used simplex search algorithm for determination of straightness error in a manufactured component from the discrete data obtained from CMM. The results confirm effectiveness of the proposed algorithm and it is computationally efficient even for large sample size problems. For obtaining exact solutions in solving non-linearity of straightness error, a new combinatorial optimization procedure is developed and presented in [39] that neglects the points inside convex hull. The results suggested that the proposed approach is robust and better than the other traditional techniques found in literature, although the results takes higher computational time. The traditional Monte-Carlo method, owing to its ability to skip the complicated calculations, was widely utilized by researchers in finding the uncertainty involved in form errors of different features [40, 41]. However, for enhancing the accuracy, a modification of traditional Monte-Carlo techniques with error ellipse theory was introduced in [42] for evaluation of straightness error and solving the uncertainty issues more precisely. The addition of error ellipse theory helps in effectively defining the positional uncertainty of sample points in form of ellipse. The comparison of experimental trials results with proposed hybrid approach for straightness evaluation have error less than 5%.

Zhu et al. proposed a defined point-surface distance function for straightness evaluation and comprises of two major phases including initial solution assessment and error calculation. The proposed computational algorithm confirm its robustness and easy to use due to its adequate control over differential translation and rotation [43]. Similarly, for accurate evaluation of straightness error, the authors proposed parameter less data envelopment analysis technique forming convex hull set as minimum zone [44]. The results proved effectiveness of the proposed procedure of data envelopment analysis having higher accuracy and takes less computational time in comparison to non-linear programming approach, LSM and optimization technique zone. Similar work is performed in [45], the authors presented a non-linear optimization method (NOM) combining the least square method with simplex search techniques for evaluation of straightness error with improved efficacy. In addition, the data filtering technique is utilized to remove the outliers for improving the performance of the proposed algorithm which is confirmed by improved results as compared to traditional techniques. The results when compared with convex hull method and LSM were proved to be reliable and more precised on higher sample points of different manufactured parts.

3.1.2 Advanced Optimization Techniques

The advanced optimization algorithm has several advantages as compared to traditional computational techniques i.e., ease of use, simplicity, flexibility etc. Due to this reason, several advanced optimization algorithm are applied by researchers for improving the accuracy in straightness error evaluation. Wang et al. applied basic genetic algorithm (GA) for evaluating straightness error obtaining improved results with precision as compared to traditional LSM method [46]. A new and improved GA is introduced in [47] for evaluating planar and spatial straightness error by solving the complex nonlinear fitness function subject to intricate constraints. For making the traditional GA more robust, a blend crossover operators is incorporated with population and offspring size taken as 20 each. The results of straightness error proved that improved GA outperformed traditional techniques efficiently. Cui et al. in their study applied classical particle swarm optimization (PSO) algorithm by changing the inertia weight as shown in below equation for evaluation of straightness error. The results for straightness error confirm that the PSO algorithm performed superior to traditional techniques and conventional GA technique [48]. Similarly, Mao et al. have applied PSO algorithm for evaluating spatial straightness error and found PSO has strong capability in solving such non-linear optimization problems effectively [49]. Furthermore, authors have utilized ant colony optimization (ACO) algorithm for straightness error assessment based on minimum zone error method. The fast convergent and precised results are advantages of ACO algorithm over conventional simplex search and Powell computational techniques in determining straightness error. The ACO global search ability make it realized enhanced results as compared to GA [50].

For further enhancing the efficiency and local search ability of classical ACO algorithm, a new improved ACO algorithm is proposed for determining straightness error by hybridizing it with local search Powell method (see Fig. 8a) [51]. The hybrid ACO-Powell algorithm have enhanced the convergence speed significantly and results proves that the proposed algorithm outperformed basic GA and other traditional techniques in providing solution to straightness error. In [52], a search algorithm based on beetles' variable step was introduced to evaluate the straightness error formulated using LSM method. The global search of beetle algorithm is enhanced by altering the step size as the iterations increased. The final results when compared with basic GA and PSO, the beetle antenna variable step algorithm provides superior solution. For evaluation of straightness error, a novel hybridization was performed combining artificial fish swarm algorithm (AFSA) with least square method. In addition, for improving the diversity and convergence speed of artificial fish swarm algorithm new mutation and elimination

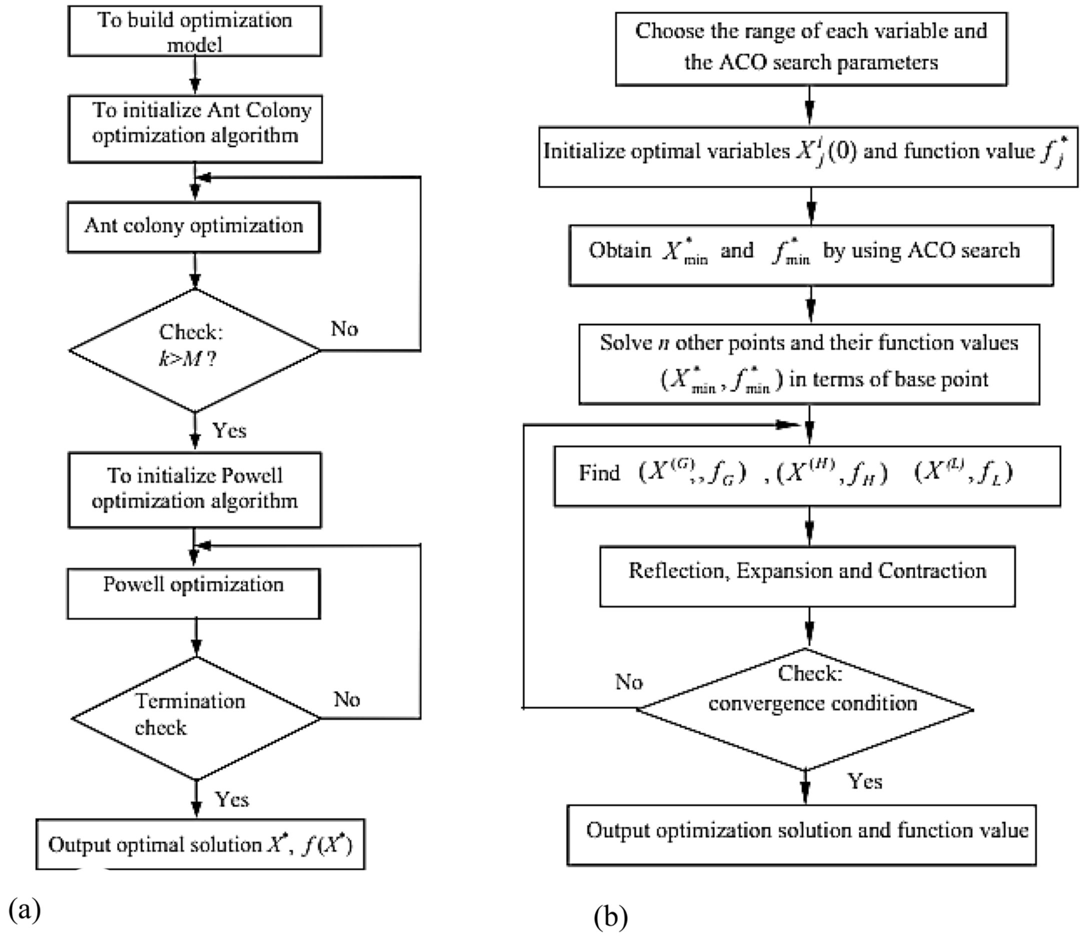

Fig. 8 Flowchart for **a** hybrid ACO-Powell algorithm [51] **b** hybrid simplex search – PSO algorithm [58]

mechanism were also added. The straightness error results were better for hybrid LSM—AFSA algorithm when compared with traditional GA [53].

The advantage of differential evolution (DE) algorithm in terms of few controlling parameters, simple structure and effective exploration ability makes it suitable candidate in metrological feature evaluation [54]. Due to such pros, the authors in [55] proposed an improved differential evolution algorithm for solving non-linear axis straightness error fitness functions. The improved DE combines opposition-based learning for better exploration and good point set method for enhanced exploitation for improving the overall performance of basic DE algorithm. When compared with past improved DE algorithm and traditional LSM method, the proposed improved DE shows superior results in terms of precised straightness error. In [56], the authors applied PSO algorithm for determining axis straightness error formulated using LSM methodology. The results concluded that the accuracy of straightness evaluation improved by nearly 25% as compared to conventional computational techniques. The fast convergence, less controlling parameters and better results are some of advantages of PSO algorithm in comparison to GA justifying the greater use of PSO for straightness error evaluation [57]. To overcome the traditional techniques problem of exploration, a hybrid simplex search—PSO algorithm (see Fig. 8b) was proposed in [58] for straightness error evaluation. The results of the hybrid algorithm not only outperformed traditional techniques such as simplex, Powell method but it has also shown higher accuracy in comparison to ACO, GA.

Authors in [59] evaluated the straightness error applying a new improved version of artificial bee colony, named as IABC, from a set of CMM data points. In IABC algorithm, first improvement was initialization of population that is based on opposition-based learning and second improvement deals with greedy selection for food source selection of employed bees. The results of IABC were superior for straightness error when compared with traditional ABC and other computational techniques in terms of convergence speed and global diversity. Recently, Hui et al. have proposed a hybrid method for straightness error evaluation by analyzing the assembly consistency factors

Table 1 Summary of computational techniques on straightness error evaluation

Computational technique	Description	Recommendation	References
Computational geometry-based technique	Traditional techniques	Computationally less complex, require short time for execution	[32]
Convex hull set		Exact solution and uniqueness in solution	[33]
Convex polygon		Computationally efficient and takes less time	[34]
Region-elimination search		Minimized the sample points and provided desired level of accuracy	[35]
Successive quadratic programming		The results are within the range of 10–3 mm	[36]
Geometry computational algorithm		Outperformed well-known sorting algorithms in terms of computational complexity	[37]
Simplex search algorithm		Computationally efficient even for large sample size	[38]
Combinatorial optimization approach		Robust but takes higher computational time	[39]
Monte-Carlo method with error ellipse theory		Successfully define the positional uncertainty of sample points and accurate results with 5% error	[42]
Distance function-based algorithm		Robustness and easy to use due to its adequate control over differential translation and rotation	[43]
Data envelopment analysis		Higher accuracy and takes less computational time in comparison to non-linear programming approach	[44]
Non-linear optimization method		Results are reliable and precised on higher sample points	[45]
Genetic algorithm (GA)	Advanced optimization algorithms	Improved results as compared to traditional LSM method	[46]
Improved GA		GA based on blend crossover operators outperformed traditional techniques efficiently	[47]
Particle swarm optimization (PSO) algorithm		PSO algorithm performed superior to traditional techniques and conventional GA technique	[48]
Particle swarm optimization (PSO) algorithm		PSO has strong capability in solving such non-linear optimization problems effectively	[49]
Ant colony optimization (ACO) algorithm		ACO global search ability make it realized enhanced results as compared to GA	[50]
Hybrid ACO-Powell algorithm		Enhanced the convergence speed significantly and outperformed basic GA and other traditional techniques	[51]
Beetles search algorithm		The proposed beetle antenna algorithm with variable step provides superior solution than GA and PSO owing to higher diversity	[52]
Artificial fish swarm algorithm (AFSA) + LSM		For improving the diversity and convergence speed of artificial fish swarm algorithm new mutation and elimination mechanism were added	[53]
Improved differential evolution algorithm		DE combines opposition-based learning for better exploration and good point set method for enhanced exploitation	[55]
Improved artificial bee colony (IABC)		Initialization of population based on opposition-based learning and second improvement deals with greedy selection of employed bees	[56]
Particle swarm optimization (PSO)		Accuracy of straightness evaluation improved by nearly 25% as compared to conventional computational techniques	[57, 58]
Hybrid simplex search – PSO algorithm		Higher accuracy in comparison to ACO, GA for straightness error	[59]
GA-multi-class support vector machine-improved Kuhn–Munkres method		Reduced the straightness and assembly consistency index to 0.08 from 0.19	[60]

combining GA optimized multi-class support vector machine and improved Kuhn–Munkres method [60]. The results prove that the proposed hybrid machine learning method can successfully minimized the straightness error thus reducing the assembly consistency index to 0.08 from 0.19 (Table 1).

3.2 Flatness Error Evaluation Using Computational Techniques

3.2.1 Traditional Techniques

In past, several researchers have utilized traditional computational techniques in determining the flatness error from

CMM extracted data of different industrial parts. Raghunandan and Rao used computational based geometric technique that follows convex hull concept for the evaluation of flatness error extracting data points realized from CMM. They also investigated the adequate location of sample points and optimal sample size of data points for evaluating accurate flatness error. They found that poor surface quality requires higher amount of data points to be sampled for accurate evaluation of flatness error [61]. Damodarsamy and Anand applied a new technique based on normal plane for evaluating the minimum zone flatness error using a set of direction cosines parameters. Then simplex search methodology was adopted to search the zone and find the exact solution of the flatness error in terms of different parameters. The results revealed that normal plane method outperformed LSM, convex hull and constrained optimization method for different size datasets in terms of flatness error [62]. Li et al. proposed a hybrid method combining reduced constraint region and convex hull concept for determining minimum zone flatness error. The former emphasis is on exploring effective enveloping plane while the latter exploit the convex hull in that particular route. When compared with several well-established traditional techniques, the reduced constraint region and convex hull hybrid algorithm outperformed all significantly for small as well as large sample size in flatness error evaluation [63].

Similarly, a geometric search approximation algorithm is presented for assessing the flatness error, based on three edge points treated as reference points and developing auxiliary and reference planes simultaneously. By comparing the distance differences, the value of flatness error were determined for different sample sizes. It was found that proposed method significantly reduced flatness error in the range of 0.3 – 1.7 μm in comparison to convex hull method and LSM method [64]. Wang et al. in their study used modified variant of gray level co-occurrence matrix (GLCM) for effectively evaluating the flatness error based on gray scale. Based on GLCM method, three different parameters were determined namely, contrast, entropy, and correlation that in turn define the surface quality and influence the flatness error [65]. Zhu and Ding proposed a new computational techniques based on equality between the inner radius of the convex hull and width of a data point set for flatness error evaluation. The proposed approach provides almost exact solution and less computationally expensive as compared to other traditional techniques [66]. For finding straightness error, Ye et al. introduced novel adaptive and iterative neighborhood-based search strategy. The proposed approach follows the development of initial datum plane using LSM and then candidate datum plane is created based on minimum value of flatness error defining the search space between new and old datum planes. The result of proposed approach provides exact value of flatness error which is comparable with other traditional techniques such as LSM and CPRS [67].

Tian et al. determined the minimum zone flatness error using region searching method from the extracted data points coordinates. The results concluded that proposed region searching technique determined flatness error value which is 5.97% better as compared to conventional LSM [68]. In order to improve the practicality, Xu et al. determined the minimum zone flatness error combining L9 orthogonal test design and area searching algorithm. The initial reference plane is created from the extracted data points and rotating coordinate system. The results found that the proposed approach are better than traditional techniques such as LSM, convex hull and computational geometry method, while the results are lesser accurate than advanced optimization algorithm. However, the proposed method takes comparatively 10 times lesser time in evaluation of flatness error as compared to advance algorithms [69]. A robust convex hull set algorithm based on computational geometry is presented in [70] for evaluating flatness error effectively, coded further and utilized in a software application for the calibration of gauges in industries.

Although, the computational efficiency of convex hull method is reasonable, however, a modified convex hull edge method (CONHEM) is proposed in [71] for enhancing the efficiency in flatness error evaluation for higher data sets. The geometrical relationship was established between two-dimensional projection and three-dimensional convex hull of an individual data point. From results, it was found that proposed CONHEM method is robust, computationally inexpensive and takes less time in flatness error evaluation as compared to different techniques such as OTZ, LAT, CPRS, COM etc. In [72], three different theorems were presented justifying the use of incomplete convex hull for the assessment of flatness error, thus reducing the computational time in finding solution. The theorems explain the removal of redundant points, solution with lesser points and possible candidate selection at convex hull edges. The results were found to be effective and shows that computational efficiency is improved significantly in determining the flatness error. Deng et al. proposed method based on valid characteristic point having rapidly contracted zone for evaluation of flatness error using minimum zone method [73]. The method deals with rapidly contracting the tolerance zone of the geometric characteristics point parameters of enveloping feathers and iterated the minimum zone value quickly. When tested on large data points, the proposed method comes out as computationally fast and outperformed other traditional algorithm in flatness error evaluation.

For the evaluation of flatness error based on CMM data, the Monte Carlo simulation method was employed in [74]. The authors developed a model considering the repeatability of the data points. The result from study confirms the effectiveness of the method that evaluates flatness error with 95% probability and validated it by optical glass results. Calvo et al. proposed a new vectorial method for evaluation of flatness error evaluation from data coordinates. The accuracy of the proposed method is satisfactory and outperforming many well-recognized algorithm in flatness error evaluation [75]. Yue et al. investigated the flatness error using measured points classification technique in three major types such as low, high and saddle. The proposed approach classify set of low points as minimum zone low points and set of high points as minimum zone high points with none of the points appear in saddle one. The results revealed ten times higher speed of computation and provides better results when compared with conventional minimum zone method [76].

3.2.2 Advanced Optimization Techniques

An improved genetic algorithm (IGA) with different offspring and population size is presented for solving the minimum zone flatness error. The proposed IGA incorporates blend crossover operator for enhancing the search capability and effective solving of separable objective functions of flatness error. From results, it was found that IGA and PSO has same results with different value of parameters while IGA outperformed traditional LSM [77]. A modified artificial bee colony (MABC) algorithm was applied for fast and accurate evaluation of flatness error by introducing tabu search and traction bees concept. The proposed modification in classical ABC enhances the convergence speed with improved quality of solutions. The flatness error obtained by MABC were 0.9 µm better than the basic ABC, GA and PSO algorithm thus establishing it effectiveness in finding better solutions [78]. Cui et al. proposed GA based method (GAM) for evaluation of different form errors including flatness error. The results proved that proposed algorithm is easy to use, computationally fast and have adequate precision. The straightness error found were better than traditional techniques such as LSM, LAT and MRS [79]. Similarly, a robust and efficient differential evolutionary (DE) algorithm was applied for computing the minimum zone flatness error. Two different data samples were considered for flatness error computation and results are compared with elitist selection-based GA (EGA) and traditional LSM. It was verified from results that DE performed superior to EGA and LSM in terms of precised computation of flatness error [80].

Recently, a novel hybrid flatness error evaluation algorithm is introduced combining the convex hull method with improved PSO. The improvement in PSO is incorporated in terms of non-linear inertia weight w and dynamic learning factors c_1, c_2 equations, which were determined after several trials. The results of flatness error based on proposed hybrid method was 44 µm better than classical PSO algorithm results [81]. Furthermore, Tseng in his study proposed GA based algorithm i.e., float encoding GA (FEGA) utilizing real encoding scheme, for flatness error evaluation. The results for flatness error shows that proposed FEGA is more effective and performed better than the traditional LSM and convex hull method [82]. Similarly, Zhang in his study also proposed hybrid optimization algorithm for evaluating minimum zone flatness error. The proposed algorithm provides hybridization of chaos optimization algorithm (COA) with Powell search method as shown in Fig. 9. Initially, the COA runs to perform global search and then Powell search utilized to exploit the good solutions found by COA. The results of flatness error proves that proposed COA-Powell search converges fast and provides more accurate results as compared to traditional methods and classical GA [83].

To improve the computational accuracy in determining flatness error, Yang et al. proposed an adaptive hybrid teaching learning-based optimization (AHTLBO) to improve the search capability of the classical TLBO algorithm. The proposed algorithm incorporates adaptive factor and hybridizes with shuffled frog leaping algorithm (SFLA) that further enhances the convergence speed and global search behavior. It was found that the best results were obtained from AHTLBO and have faster convergence in comparison to basic TLBO, PSO, GA and LSM [84]. Zhang and Luo in their aim to further minimize the flatness error introduced a similar hybridization method combining Powell search and artificial fish swarm algorithm (AFSA) (see Fig. 9). The proposed algorithm improved the exploitation behavior of basic AFSA algorithm using local search Powell method and thus improved the precised flatness error results. It was evident from results that proposed approach reaches to exact flatness error value at higher speed in comparison to GA, PSO and simplex search [85].

In recent times, Miko in his study employed regression analysis based on partial point sets for computing the flatness error using finite number of data points. The results concluded that regression analysis method effectively predict the flatness error for different size of data points by taking help from extrapolation and predicting with higher regression coefficient nearly close to 1 [86]. Pathak and Singh in their study investigated various form errors including flatness using improved version of PSO algorithm named as MPSO. The MPSO generates new swarm positions and fitness solution using novel search equation. In addition, a greedy selection mechanism was incorporated for best position selection based on fitness solution. The results of flatness error, were better in comparison to GA and basic PSO, proves the effectiveness of the proposed MPSO in overcoming exploitation drawback of classical PSO algorithm [87]. Yu and Huang in their study presented an improved variant of PSO by introducing genetic hybrid gene, named as GHPSO accordingly,

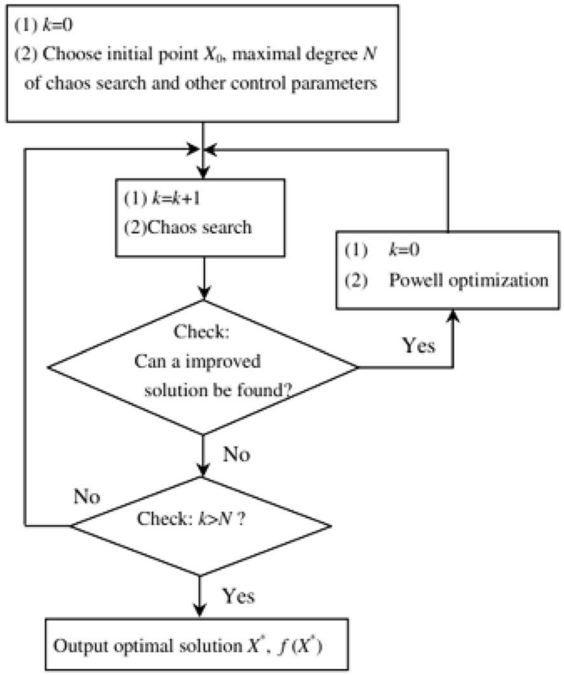

Fig. 9 Hybrid COA-Powell search method procedure [83] and AFSA-Powell search [85]

for flatness error evaluation. The improvement is realized by adding the crossover and mutation operators of GA in PSO algorithm. The results proved that proposed algorithm is simple, efficient, and provided more accurate results as compared to LSM, GAM, PSO [88].

A real coded efficient GA (EGA) is presented for flatness error measurement and evaluation. In EGA, an elitism operator is introduced replacing the roulette wheel selection and prevents the loss of good quality solutions. The results when compared with literature shows that the proposed EGA has higher accuracy, precision, and repeatability [89]. Abdulshahed et al. applied cuckoo search (CS) optimization algorithm for flatness error computation and the results are compared with PSO, Convex hull method, and LSM procedure. The CS algorithm based on levy flights were proposed by Yang [90] is unique and has strong capability in solving non-linear optimization problems. The results for different data points proved that CS algorithm comprehensively outperformed PSO and convex hull method in flatness error evaluation of manufactured parts [91]. Jiang et al. proposed a new rotation method based on GA for determining the flatness error. The rotation angles of measured points were considered as parameters to be optimized using GA for minimum zone straightness. It was found that proposed approach with GA can efficiently determine the flatness error with ease [92] (Table 2).

3.3 Circularity Error Evaluation Using Computational Techniques

3.3.1 Traditional Techniques

In past two decades, several work have been performed utilizing traditional computational methods in evaluating circularity error for curved features in manufactured parts. A linear approximation technique (LAT) was proposed for various form error determination including circularity in. The LAT requires larger time for computation of results, however, it has provided improved results, reduces complexity when compared with LSM and MRS method [93]. Venkaiah and Shunmugam in their study presented computational geometry technique for evaluation of circularity error from the measured profile data points. The proposed technique involves the use of convex hull and have the advantage of easy to follow with adequate visualization at each iteration. From results, it was revealed that proposed method perform superior in terms of fast and accuracy in circularity evaluation in comparison to simplex search technique [94]. Dhanish in his work proposed a new algorithm based on combination of coordinate point transformation and Chebyshev approximation for computing the circularity error. The algorithm is coded in C + + and results depicted that proposed hybrid method provides accurate value of circularity error with maximum and minimum value as 0.111857 and −0.11195 respectively [95].

Huang et al. in their study applied a new concept based on area hunting method for specific arrangement of the measured data points in circularity evaluation. It was found that the area hunting method outperformed traditional LSM in terms of precised circularity value improved by 4.16% [96]. Zhu et al. used an effective steepest descent algorithm for determining circularity value based on ANSI and ISO standards. The steepest descent concept deals with finding minimum translation distance between to convex polygons also LSM is used for providing initial guess points. The results of the circularity error proves the robustness and high precision of the steepest descent algorithm when compared with other techniques in literature. Also, the computational time increases as the number of sample data increases [97]. Rajagopal and Anand in their study applied selective data partition method for computing the circularity tolerance from the data extracted from CMM as per ANSI norms. The method follows the concept of fitting concentric circles utilizing the basis points. The basis points are determined by bifurcating the measured points into four quadrants. The results confirm the efficiency of selective data partition method over Voronoi method and LSM in terms of accuracy and efficiency for higher value of data sets [28]. Xiuming and Zhaoyao in their work proposed curvature method for evaluating roundness error to process CMM data points. The results were better than literature techniques which was based on

Table 2 Summary of computational techniques on flatness error evaluation

Computational technique	Description	Recommendation	References
Computational based geometric technique	Traditional techniques	Poor surface quality requires higher amount of data points to be sampled for accurate evaluation of flatness error	[61]
Normal plane method		Outperformed LSM, convex hull and constrained optimization method for different size datasets	[62]
Hybrid reduced constraint region and convex hull concept		Outperformed traditional algorithms significantly for small as well as large sample size in flatness error evaluation	[63]
Geometric search approximation algorithm		Flatness error in the range of 0.3—1.7 µm in comparison to convex hull method and LSM method	[64]
Gray level co-occurrence matrix (GLCM)		Three factors i.e., contrast, entropy, and correlation define the surface quality and influence the flatness error	[65]
Approximate minimum zone method		Easy for implementation, provides exact solutions and computationally efficient	[66]
Adaptive and iterative neighborhood-based search strategy		Provides exact value of flatness error which is comparable with other traditional techniques such as LSM and CPRS	[67]
Region searching method		Flatness error value is 5.97% better as compared to conventional LSM	[68]
Hybrid orthogonal design with area searching algorithm		Takes comparatively ten times lesser time in evaluation of flatness error as compared to advanced algorithms	[69]
Convex hull set algorithm		Utilized in a software application for the calibration of gauges in industries	[70]
Modified convex hull edge method		Method is robust, computationally inexpensive and takes less time in flatness error evaluation as compared to OTZ, LAT, CPRS, COM	[71]
Incomplete convex hull method		Results were found to be effective and improved computational efficiency achieved	[72]
Valid characteristic point having rapidly contracted zone		Computationally fast and outperformed other traditional algorithm for larger data points	[73]
Monte Carlo simulation		Evaluates flatness error with 95% probability and validated it by optical glass results	[74]
Vectorial method		Good accuracy and outperformed well-recognized algorithm	[75]
Points classification technique		Ten times higher speed of computation and better results as compared to conventional minimum zone method	[76]

Table 2 (continued)

Computational technique	Description	Recommendation	References
Improved Genetic Algorithm (IGA)	Advanced optimization algorithms	IGA incorporates blend crossover operator and outperform LSM	[77]
Modified artificial bee colony (MABC) algorithm		Flatness error obtained by MABC were 0.9 µm better than the basic ABC, GA and PSO algorithm	[78]
GA based method (GAM)		Easy to use, computationally fast and have adequate precision	[79]
Differential evolutionary (DE) algorithm		Superior to EGA and LSM in terms of precised computation of flatness error	[80]
Convex hull method with improved PSO		Results of flatness error based on proposed hybrid method was 44 µm better than classical PSO algorithm results	[81]
Float encoding GA (FEGA)		More effective and performed better than the traditional LSM and convex hull method	[82]
Chaos optimization algorithm (COA) with Powell search		COA-Powell search converges fast and provides more accurate results as compared to traditional methods and classical GA	[83]
Adaptive hybrid teaching learning-based optimization (AHTLBO)		Adaptive factor and hybridizes with shuffled frog leaping algorithm (SFLA) provides fast convergence	[84]
Powell search and artificial fish swarm algorithm (AFSA)		Reaches to exact flatness error value at higher speed in comparison to GA, PSO and simplex search	[85]
Regression analysis based on partial point sets		Effectively predict the flatness error for different size of data points with regression coefficient close to 1	[86]
Modified PSO		New search equation and greedy selection was incorporated that outperform GA and basic PSO	[87]
Genetic hybrid gene PSO		Simple, efficient, and provided more accurate results as compared to LSM, GAM, PSO	[88]
Efficient GA (EGA)		EGA has higher accuracy, precision, and repeatability	[89]
Cuckoo search (CS)		CS algorithm based on levy flights outperformed PSO and convex hull method in flatness error evaluation	[91]
Rotation method based on GA		Efficiently determine the flatness error with ease	[92]

finding curvature radius of outer and inner convex hulls [98] (Fig. 10).

Ding et al. presented semi-definite programming for minimum zone circularity evaluation as a constrained optimization problem. In the framework of semi-definite programming, the interior point method was employed to solve the higher difficulty level and the programming complexity is lower. The results on different data sets confirms superiority of proposed method and efficient in determining roundness error [99]. Cui et al. proposed a new iterative neighborhood search approach (INSA) for computing circularity error from CMM extracted data based on minimum zone. The step of proposed approach deals with finding initial position and size of the searching scope. The next step is determining datum centers along with center and radius of searching area following evaluation of circularity error. The results recommended that for 100 data points the time taken to search the solution is comparatively better (nine times) and provides accurate circularity value [100]. The use of convex hull method is common in determining different form error in various literature. To this end, Xiuming and Zhaoyao extend the use of convex hull method for evaluating circularity error along with coordinate transmission. Two different data sets from CMM are considered to test the proposed approach and concluded that convex hull method based on polar coordinated can efficiently evaluate the roundness error [101]. Gadelmawla in his study introduced a novel computational geometric technique for evaluation of circularity error. The technique emphasis on finding three points for developing two features i.e., a circle and a point online connecting first and third point. The point sequence direction was found out based on second point location factor. When compared with digital instrument, the result of circularity by proposed approach doesn't vary more than ± 2.27% with effective reduction in computation time [102]. Lei et al. proposed a novel geometry based Geometric Approximating Searching Algorithm (GASA)

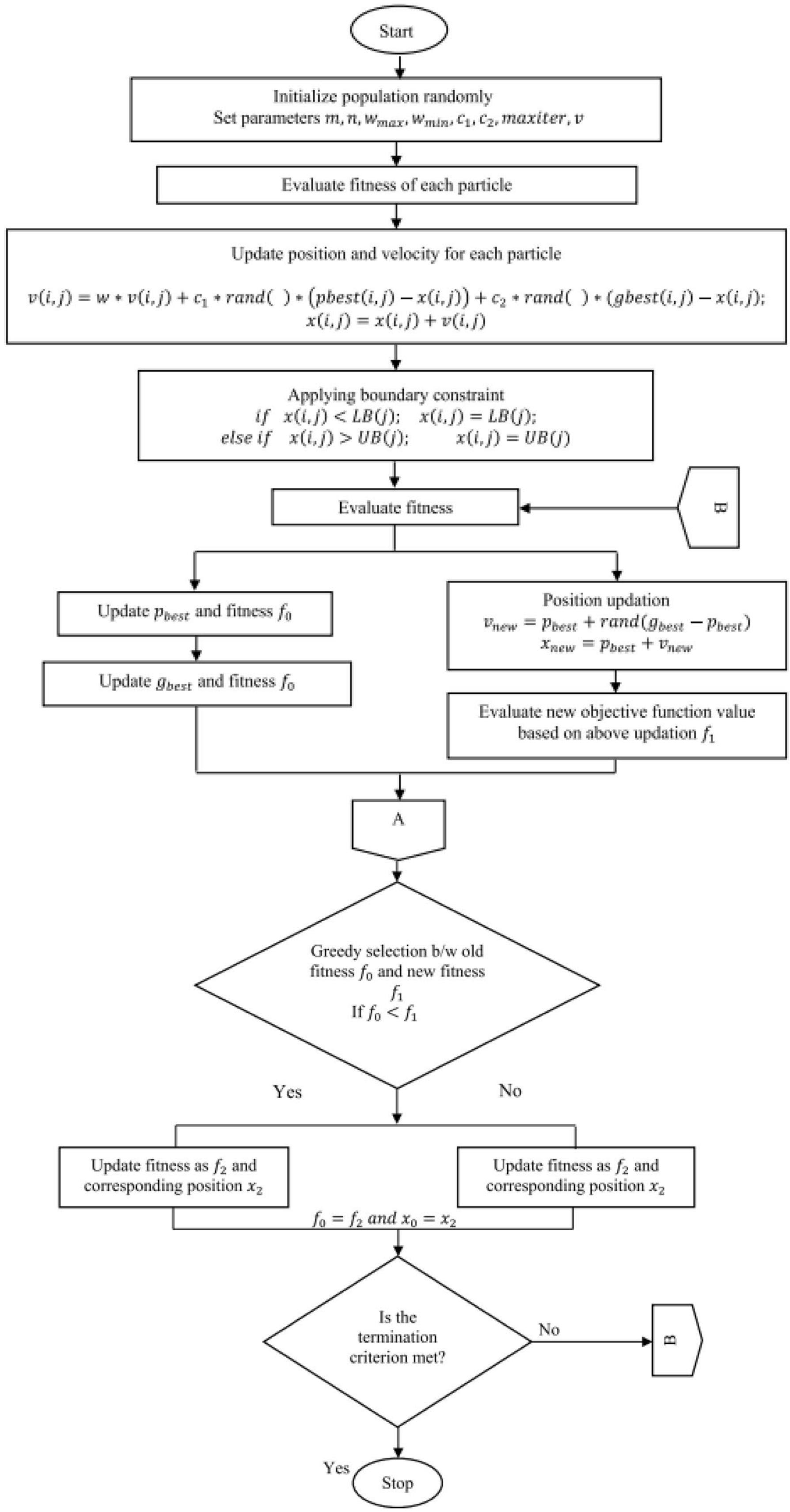

Fig. 10 Steps of MPSO algorithm for flatness error evaluation [87]

for evaluating circularity error effectively. The proposed algorithm is shown in Fig. 11, which begins with allocating initial reference point of a square, then following the radius coordinates of the vertex of square and finally evaluating the radial extreme distance for circularity error. It was found that the proposed GASA is easy to use and efficiently determine the minimum zone circle circularity error [103].

Jiang et al. presents a novel algorithm based on minimizing the uncertainty in fitting data points by following the concept of profile confidence for evaluation of circularity error. The results revealed that the computed circularity value is closer to actual value and provides significant accuracy owing to its ability to create actual round profile [104]. Similarly, Lei et al. proposed polar coordinate transform algorithm (PCTA) by calculating polar coordinates of circular region around least square circle for evaluating circularity error. The results proved that accuracy of PCTA in evaluating circularity error depends on two different parameters and higher value generally provided greater accuracy [105]. Li et al. evaluated circularity error using the integration of α-hull with the Voronoi diagram. The vertices of the Voronoi diagram by α-hull having minimum radius separation. The results showed higher efficiency of proposed approach in solving minimum zone circularity as compared traditional convex hull method [106].

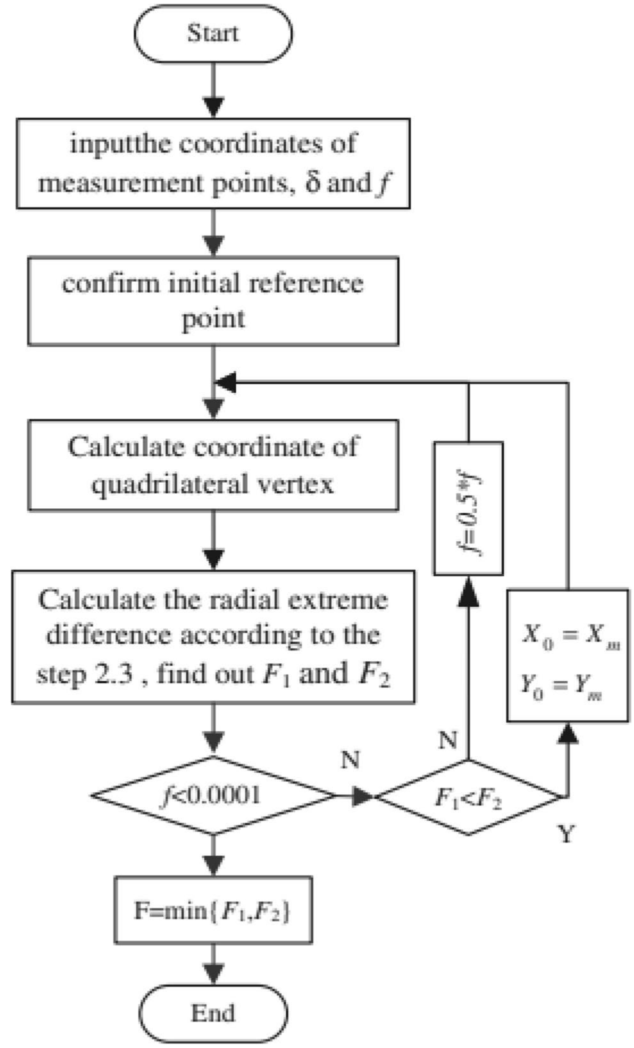

Fig. 11 Flowchart of GASA for determining circularity error [103]

3.3.2 Advanced Optimization Techniques

Several authors have used advanced optimization algorithms for fast and accurate evaluation of circularity error, that will directly influence the life and performance of industrial and mechanical products. Chen et al. in their study proposed a hybrid method combining simulated annealing (SA) and Hooke–Jeeves pattern search for circularity error evaluation. The SA algorithm is better in exploration however, to enhance the exploitation capability it was hybridized with local search Hooke–Jeeves pattern search method for balancing the diversity in hybrid algorithm. The results in determination of circularity error in Geneva cam and gear revealed that proposed hybrid approach have reasonably good accuracy and computationally inexpensive [107]. Wen et al. introduced an effective GA for computing circularity error without need of parameters such as crossover and mutation, that are mandatory requirement in classical GA. The results proved that efficient GA is taking lesser computational time (almost half) and accurate results (almost half) in comparison to classical GA for solving circularity error problem [108]. Du et al. proposed novel PSO algorithm by changing the inertia weight value and attaining its best value, for evaluation of minimum zone circularity error. The linearly changing inertia weight PSO results are compared with PSO having three different values of inertia weight i.e., 0, 0.5 and 1. From results, it was found that inertia weight decreasing linearly from 0.9 to 0.4 provides best results. The PSO based on inertia weight found superior to evaluate accurate circularity error as compared to GA and LSM [109]. Sun in his study proposed five new variants of PSO altering the inertia weight, number of swarms and maximum velocity for computing the circularity error. The results on determining floppy disk circularity error were collected for five variants of PSO and it was found that PSO with combination of maximum velocity and inertia weight provide accurate results than all other PSO variants and GA [110].

Kumar et al. proposed parameter less TLBO algorithm for circularity error measurement and the results are compared with classical PSO algorithm. When comparing both advanced optimization algorithms in terms of accuracy and convergence time, it was found that both obtained similar results while TLBO takes higher computational time in comparison to PSO [111]. To solve the computationally expensive minimum zone tolerance circularity error efficiently, a fast GA is presented by altering five different parameters, namely, mutation, crossover, search space, population size and end condition, of classical GA. The results of proposed GA are compared on seven different samples for determining circularity error with classical GA and EGA results. The results revealed that the computation time significantly reduced by selecting optimal GA parameters and that provides greater accuracy making it suitable candidate for on-line inspection [112]. Jin et al. in their work found circularity error based on classical DE algorithm on two different data sets extracted from CMM. The results of DE circularity error were found to be similar to GA while it shows 10% improvement as compared to LSM [113]. Sinivasu and Venkaiah presented a hybrid method that employs LSM with novel probabilistic global search Lausanne (PGSL) technique for circularity evaluation based on minimum zone. The update procedure of hybrid algorithm balances the global and local search effectively for determining optimal solutions in search space. The results on three different datasets show that proposed approach have better accuracy in terms of circularity error as compared to LSM results [114].

Pathak and Singh in their study applied constriction factor PSO (CFPSO) algorithm for evaluation of different form errors including minimum zone circularity. The constriction factor is added in the swarms velocity updation equation for enhancing the exploration in initial iterations and lower value of this factor at higher iterations will enhance the exploitation behaviour of proposed algorithm. The CFPSO algorithm provides nearly 25% improvement in circularity error when compared with traditional techniques such as LAT, OTZ, LSM and GA and also have faster convergence [115]. Rossi and Lanzetta in their study applied a metaheuristics such as GA for evaluation of circularity error within a given search space. Several datasets were utilized to calculate the circularity error, GA shows higher datasets incurred higher accuracy but also results in higher computational time [116]. Meo et al. in their work established a relationship among search space, data set size, inspection time and convergence speed in determination of minimum zone circularity evaluation using classical GA. It was found from results that lower size of search space, higher datasets results in accurate evaluation of circularity error however, inspection time increases with higher datasets [117]. Ming et al. in their study performed circularity evaluation by hybridizing AFSA with geometric algorithm for enhancing the diversity of basic AFSA algorithm and performance in determining optimal solution in search space. When compared with classical GA and AFSA, the results for circularity error were more precised with enhanced convergence achieved [118] (Table 3).

3.4 Cylindricity Error Evaluation Using Computational Techniques

3.4.1 Traditional Techniques

The cylindricity error determination is imperative in maintaining precision of assembled and mechanical parts, that finally effect the wear rate among components and assembly accuracy. Several authors have used traditional computational techniques for evaluating minimum zone cylindricity error effectively. Lei et al. applied a new geometry optimization searching algorithm (GOSA) for evaluating cylindricity error, their work showed that the result of proposed approach depends on a pre-set factor δ. The higher the value of δ, the more accuracy can be realized through GOSA, δ with 0.0001 mm provides minimum cylindricity error [119]. Venkaiah and Shunmugam extended the use of computational geometric technique for creation of limacon cylinder in accurate evaluation of cylindricity error. The technique work on extreme points as determined by the development of convex hulls. The proposed algorithm is computationally fast and provides higher accuracy in finding solutions of cylindricity error [120]. Zhu and Ding in their work presented a new algorithm and explored the cylindricity error evaluation problem centered on kinematic geometry, deriving the signed point-to-surface distance function with its increment. The results on different CMM datasets found that proposed algorithm was computationally efficient and global solution realized owing to analytical initial solution [121].

Recently, authors have proposed an adaptive variational body approach to solve the minimum zone cylindricity problem from data points. The computational approach largely followed geometric transformation strategy for developing the cylindricity models. The results proved that proposed method outperform traditional LSC method by 4% in cylindricity evaluation and have faster speed of computation [122]. Zheng et al. suggested a novel kinematic geometry optimization algorithm (KGOA) for cylindricity error solution, in which extraction of points for individual feature using convex set approach and projective transformation. The KGOA effectively determine the value of cylindricity error with half computation time in comparison to CMM software results [123]. The measurement and evaluation of cylindricity error was performed in studies of Liu et al. by utilizing four- and five-point separation techniques. These studies focused on creation of high precision cylindrical profiles that may be extended to in-situ measurement. The studies also adopted monte Carlo simulation method for evaluation of cylindricity error [124, 125]. Recently, Liu et al. further proposed a new method based on minimax concept for solving problem of cylindricity error. The minimax model is further linearized using Taylor expansion and used for approximating the cylindricity error. The results of cylindricity error were in accordance with MZC, five-point method and other published techniques [126].

Based on ISO norms, Liu et al. suggested a new method based on increment-simplex algorithm for cylindricity error evaluation. The proposed method followed incremental steps in removing the redundant points or the points having less influence on the cylindricity evaluation. The results of their work recommended that increment-simplex algorithm provided accurate results for MZC and MIC with faster computation of cylindricity error [127]. Zheng et al. proposed two different compensation techniques i.e., Slope and helix compensation method for accurate extraction and online measurement of cylindricity error. In addition, an improved simplex algorithm used for solving non-linear cylindricity problem. The results are in accordance with the experimental results [128]. Lao et al. presented a new hyperboloid method for determining efficiently cylindricity error not by creation of minimum zone directly but in the form of iterative hyperboloids zone with initial cylindrical axis assumed as vertical. The results showed that the cylindricity realized with proposed approach was nearly half than conventional LSC method [129].

3.4.2 Advanced Optimization Techniques

For determining and solving the non-linear minimum zone solution of cylindricity error, several work have been introduced in past applying advanced optimization algorithm for guarantee global minimum solution at a faster rate than traditional techniques. Lai et al. suggested genetic algorithm (GA) for effective evaluation of cylindricity error. For increasing the efficiency of GA, several simulation trials were performed for choosing best value of population size and mutation operator. The GA approach provides superior results than least square method in terms higher accuracy and efficiency, for cylindricity error [130]. Yang et al. proposed a new and improved variant of harmony search algorithm named as IHS for evaluation of cylindricity error. The basic harmony search algorithm mimics the creation of music and its evaluation for achieving the optimization goal

Table 3 Summary of Computational techniques on Circularity error evaluation

Computational technique	Description	Recommendation	References
Linear approximation technique (LAT)	Traditional computational techniques	Improved results with reduced complexity when compared with LSM and MRS	[93]
Computational geometry technique		Computationally fast and accurate in circularity evaluation in comparison to simplex search technique	[94]
Hybrid coordinate point transformation and Chebyshev approximation		Provides accurate value of circularity error with maximum and minimum value as 0.111857 and -0.11195 respectively	[95]
Area hunting method		Outperformed traditional LSM in terms of precised circularity value improved by 4.16%	[96]
Steepest descent algorithm		High precision when compared with other techniques and higher computational time with higher sample size	[97]
Selective data partition (SDP) method		Higher efficiency over Voronoi method and LSM in terms of accuracy	[28]
Curvature method		Results were better than literature traditional techniques	[98]
Semi-definite programming		Superior and efficient in determining roundness error	[99]
Iterative neighborhood search approach (INSA)		For 100 data points the time taken to search the solution is comparatively better (nine times) and provides accurate circularity value	[100]
Convex hull algorithm		Convex hull method based on polar coordinated can efficiently evaluate the roundness error	[101]
Computational geometric technique		In comparison of digital instrument, its accuracy doesn't vary more than ± 2.27% with effective reduction in computation time	[102]
Geometric Approximating Searching Algorithm (GASA)		Easy to use and efficiently determine the minimum zone circle circularity error	[103]
Method based on reducing uncertainty in fitting data points		Significant accuracy owing to its ability to create actual round profile	[104]
Polar coordinate transform algorithm (PCTA)		Depends on two parameters and their higher value provides greater accuracy	[105]
α-hull with the Voronoi diagram		Higher efficiency of proposed approach in solving minimum zone circularity as compared traditional convex hull method	[106]

Table 3 (continued)

Computational technique	Description	Recommendation	References
Hybrid simulated annealing (SA) and Hooke–Jeeves pattern search	Advanced optimization algorithms	Reasonably good accuracy and computationally inexpensive	[107]
Effective GA		Flatness error obtained by MABC were 0.9 µm better than the basic ABC, GA and PSO algorithm	[108]
Improved PSO method		Superior to evaluate accurate circularity error as compared to GA and LSM	[109]
New variants of PSO		Provide accurate results than all other PSO variants and GA	[110]
TLBO algorithm		Obtained similar results while TLBO takes higher computational time in comparison to PSO	[111]
Fast GA with five different variation		Results revealed that the computation time significantly reduced by selecting optimal GA parameters and it provides greater accuracy	[112]
Classical DE algorithm		DE circularity error were found to be similar to GA, while it shows 10% improvement as compared to LSM	[113]
Hybrid LSM with novel probabilistic global search Lausanne (PGSL)		Better accuracy in terms of circularity error as compared to LSM results	[114]
Constriction factor PSO (CFPSO) algorithm		Provides nearly 25% improvement in circularity error when compared with traditional techniques such as LAT, OTZ, LSM and GA and also have faster convergence	[115]
Genetic Algorithm (GA)		GA shows higher datasets incurred higher accuracy but also results in higher computational time	[116]
Genetic Algorithm (GA)		Lower size of search space, higher datasets results in accurate evaluation of circularity error however, inspection time increases with higher datasets	[117]
Hybrid AFSA with geometric algorithm		More precised with enhanced convergence achieved as compared to basic GA and AFSA	[118]

[131]. For enhancing the convergence rate and accuracy of classical harmony search algorithm, chaos-based initialization is provided first, then a new dynamic factor is introduced to maintain the diversity and finally cauchy mutation increase the search space solution quality. The results concluded that IHS algorithm have higher accuracy than basic HS, also the improved HS converges to optimal solution in 48 iterations as compared to 285 iterations taken by classical HS [132]. Wen et al. proposed quasi PSO (QPSO) for finding the solution of non-linear cylindricity error. The results revealed that the exploration capability of QPSO make it suitable for enhanced searching in design space of cylindricity error. The results on four different datasets shown that QPSO precisely determined the cylindricity error with improved efficiency [133].

Similarly, classical PSO algorithm have been used by different authors for determining the uncertainty involved in cylindricity error evaluation and knowing the effect of cylindricity on position errors using L-F functions. The results showed higher convergence speed and greater accuracy for PSO as compared with GA and improved GA, and position error will be hampered with increase in cylindricity error [134, 135]. Li et al. proposed a hybridization of greedy sine cosine algorithm with differential evolution (HGSCADE) to develop a robust algorithm for cylindricity error evaluation. The present hybridization deals with adding opposition-based learning initialization, greedy search and DE with levy flight for local search that enhances the performance of basic sine cosine algorithm by overcoming the pros of exploitation behaviour and premature convergence. The results based on statistical analysis revealed the superiority of HGSCADE algorithm over whale optimizer algorithm, ABC, SCA and SSA in terms of convergence speed and accuracy [136]. For using hybrid methods in evaluation of cylindricity error, Luo et al. explored the combination of ABC algorithm with Tabu search for enhancing the exploration capability of basic ABC algorithm. The results when compared with GA and ACO algorithm found to be accurate with faster convergence rate of 1.2 s only [137]. Wu et al. introduced a new PSO algorithm based on comprehensive learning (CLPSO) for improving the intensification in basic PSO and applied it to compute the cylindricity error. The local search of CLPSO were performed using Latin hypercube sampling method. The solution of cylindricity error proves that proposed algorithm has fine search ability and is better in comparison on basic PSO [138].

Similarly, multi-population genetic algorithm was proposed in [139] for cylindricity evaluation. The results showed that proposed algorithm enhances global search capability and found better results with fast convergence in comparison to basic GA. Lee et al. presented support vector machine learning approach in replacing conventional LSM technique for evaluating cylindricity error. The proposed method converts the non-linear constraints into linear constraints for obtaining the solution. Supported by statistical analysis, it was found that proposed approach outperform non-linear programming and LSM in terms of CPU time and accuracy for higher number of datasets [140]. Zhang et al. introduced a hybrid algorithm for evaluation cylindricity error combining the PSO and Chaos search method. The hybridization was imperative in terms of reducing the design variable search space and thus improves the chances of realizing optimal solutions. In comparison to LSM and basic PSO, the presented hybrid method provides better efficiency and accuracy in error evaluation [141]. Similarly, Chen et al. applied basic GA for evaluating cylindricity error in engine cylinder bore. The results suggests that basic GA found minimum zone cylindricity value with more accuracy as compared to other traditional algorithms [142]. Peng and Lu in their study proposed a hybrid memetic computing algorithm following hierarchical PSO (HPSO) and latin hypercube sampling (LHS) method. The LHS method aid basic PSO in improving it search capability and also mutation operation is introduced for maintaining the diversity in hybrid approach. The results when compared with improved GA, PSO and DE algorithms found more accurate for cylindricity error [143] (Table 4).

3.5 Sphericity and Conicity Error Evaluation Using Computational Techniques

3.5.1 Traditional Techniques

For adhering to stringent dimensional and geometrical control, past studies have explored different computational techniques for determining sphericity and conicity error. In the same context, Samuel and Shunmugam in their study employed computational geometric techniques for developing convex hulls and evaluated sphericity with reference to limacoid. The proposed method have similar computational complexity when compared with other methods in literature and have unique solution with faster computation time [144]. Xianqing et al. investigated the solution of cylindricity error using new geometry optimization searching algorithm (GOSA). The steps of GOSA includes establishment of initial reference with determination of initial error followed by regular hexagon development for calculating maximum difference of radius. Finally, the sphericity error is computed by comparison of maximum radius and initial error. It was found that the initial reference point value has strong affect on the convergence performance. From results, the proposed method provides accurate, fast convergence and depends on pre-set factor δ. The lower value of factor δ gives most accurate results [145]. Wang et al. proposed a novel minimum radial separation (MRS) sphere method for modelling and evaluating sphericity error. For testing of MRS method, several datasets were considered, the results showed that computation time is a function of data sets. In addition, it provides accurate sphericity error value with minimum computation time [146]. He et. al. determined conicity error based on sequential quadratic programming (SQP) method and signed distance function. The results showed that proposed approach based on SQP have greater accuracy and better computational time in comparison to LSM [147].

Zhang et al. proposed a primal dual interior method for determining form errors in spherical and conical components, also arch search technique was utilized in recursive manner for solution of these errors. From results, it was found that proposed method computes better results and applied less computational effort for global optimal solution in comparison to SQP and heuristic method [148]. In the context of reducing computational complexity, Liu et al. introduced a new intersection chord method in computation of sphericity error. The characteristic points are replaced by the intersected chords properties thus reducing the complexity involved. It was revealed from results that though the sphericity error determined were similar to the published methods, however significant reduction in computation time was achieved [149, 150]. Mei et al. proposed a novel method based on asymptotic search for evaluation of minimum zone sphericity error. The proposed method develops a search sphere model for determining quasi- minimum zone sphere center considering least square sphere center as initial reference. The results showed that proposed asymptotic search method evaluated sphericity error accurately and efficiently for large number of extracted datasets [151].

In recent times, Zheng proposed a branch and bound (B&B) algorithm for computation of sphericity error. The method is based on realizing the minimum radial difference for square domain as lower bound, that increases further as the domain is further sub-divided. The domain thus divided will only considers the center of concentric spheres making lower bound equal to radial difference. Four different datasets were considered for testing the proposed algorithm and it was found that B&B algorithm is computationally fast, effective and provides exact results [152]. Prisco and Polini presented a new method based on transformation matrices for evaluation of sphericity and some other form errors. The coordinates extracted from CMM were transformed and best fit was realized considering the actual profile. The transformation matrices also considered the invariance of least square sum and found effective results. The results proved

Table 4 Summary of Computational techniques on Cylindricity error evaluation

Computational technique	Description	Recommendation	References
Geometry optimization searching algorithm (GOSA)	Traditional computational techniques	The higher the value of δ, the more accuracy can be realized through GOSA, δ with 0.0001 mm provides minimum cylindricity error	[119]
Computational geometry technique		Computationally fast and provides higher accuracy	[120]
Kinematic geometry method		Computationally efficient and global solution realized owing to analytical initial solution	[121]
Adaptive variational body approach		Outperform traditional LSC method by 4% in cylindricity evaluation and have faster speed of computation	[122]
Kinematic geometry optimization algorithm (KGOA)		Half computation time in comparison to CMM software results	[123]
Four- and five-point separation techniques		Creation of high precision cylindrical profiles that may be extended to in-situ measurement	[124, 125]
Minimax concept		Cylindricity error were in accordance with MZC, five-point method and other published techniques	[126]
Increment-simplex algorithm		Accurate results for MZC and MIC with faster computation of cylindricity error	[127]
Slope and helix compensation method		Improved simplex algorithm used for solving non-linear cylindricity problem. The results are in accordance with the experimental results	[128]
Hyperboloid method		Cylindricity realized with proposed approach was nearly half than conventional LSC method	[129]
Genetic algorithm (GA)	Advanced optimization algorithms	Superior results than least square method in terms higher accuracy and efficiency, for cylindricity error	[130]
Improved harmony search (IHS) algorithm		IHS algorithm have higher accuracy than basic HS, also the improved HS converges to optimal solution in 48 iterations as compared to 285 iterations taken by classical HS	[132]
Quasi PSO (QPSO)		QPSO precisely determined the cylindricity error with improved efficiency	[133]
PSO with L-F function		Higher convergence speed and greater accuracy as compared with GA and improved GA	[134]
Greedy sine cosine algorithm with differential evolution (HGSCADE)		Superior results over whale optimizer algorithm, ABC, SCA and SSA in terms of convergence speed and accuracy	[136]
ABC algorithm with Tabu search		Results when compared with GA and ACO algorithm found to be accurate with faster convergence rate of 1.2 s only	[137]
Comprehensive learning based PSO (CLPSO)		Fine search ability and is better in comparison on basic PSO	[138]
Multi-population genetic algorithm		Global search capability and found better results with fast convergence in comparison to basic GA	[139]
Support vector machine learning approach		Outperform non-linear programming and LSM in terms of CPU time and accuracy for higher number of datasets	[140]
Hybrid PSO and Chaos search		Provides better efficiency and accuracy in error evaluation	[141]
Traditional GA		Found minimum zone cylindricity value with more accuracy as compared to other traditional algorithms	[142]
Hybrid hierarchical PSO (HPSO) and latin hypercube sampling (LHS)		Results when compared with improved GA, PSO and DE algorithms found more accurate for cylindricity error	[143]

that proposed method found quick and accurate results when compared with other traditional methods [153]. Soman et al. proposed a new selective zone search technique for evaluation of minimum zone sphericity error. The method deals with determining five extreme points for development of inner and outer minimum zone spheres. The results on four different models were considered for validating the proposed approach. The results showed that proposed method provides accurate values and takes less computation time in comparison to non-linear optimization technique [154].

The conicity error was earlier determined by Chatterjee and Roth using Chebyshev approximation that defines best fit cone when only axis or vertex points are provided. The result of proposed method showed 0.0032 as conicity error in comparison to 0.0035 by LSM [155].Lei et al. further used branching and approximation algorithm for evaluating the sphericity error in balls. The method suggest creation of cube of some length around initial reference point, thus determining the length from center of sphere to vertex of cubes. The comparison provides the new reference point of new cube recursively. The results revealed that proposed method provides same result as traditional technique while taking only 0.12 s [156]. Lei et al. applied geometrical optimization searching algorithm (GOSA) for conicity error evaluation. The method of GOSA is quite simple and already discussed in cylindricity error evaluation and results showed conicity error results were better than CMM results [157].

3.5.2 Advanced Optimization Techniques

The sphericity and conicity errors have significant influence on the functional performance of rotational components, thus several advanced optimization algorithms were applied for improved evaluation of these errors. Wen and Song proposed an immune evolutionary algorithm (IEA) that mimics the defense mechanism of immunity system for evaluation of sphericity error. The solution of sphericity error were determined by EGA, IGA and IEA on three different datasets. The results showed that IEA performed better than IGA in accuracy and takes only 2.2 s in comparison to 2.5 s and 8.6 s by IGA and EGA respectively, in evaluating sphericity error [158]. Wen et al. further used PSO algorithm for determining conicity error where each point is represented by an individual particle. When compared with IEA and GA on different datasets, the PSO algorithm provides improved conicity error value [159]. Xiulan et al. in their study proposed an improved GA for evaluating minimum zone sphericity error. The improved GA involves real-coded floating-point representation for each optimized variable. In addition, blend crossover operator was also incorporated in the basic GA procedure. The results showed improved GA takes less computation time and provide accurate error value of 0.00967 which is significantly smaller than the minimum zone solution (0.0132) [160].

Rossi et al. proposed a worst-case method for determining sphericity error which was assessed by heuristics such as GA, PSO, ACO and the results are compared with traditional LSM. The results on different datasets showed that proposed parameters of GA successfully evaluated and sphericity error, outperformed PSO, ACO and LSM [161]. Huang et al. in their study proposed a hybrid method combining heuristic search algorithm with feature points model (HAS-FPM) for effective evaluation of sphericity error. It was revealed from results that the proposed approach improved solution by nearly 60% in comparison to intersecting chord method, energy method and IEA. In addition, the computation time by proposed method is 0.01 s only, while other method takes a higher order of time [162]. Xuyi and Ming presented the application of whale optimization algorithm (WOA) in evaluating the minimum zone sphericity error. The results when compared with IEA, EGA and IGA, the WOA provides better results with comparable accuracy in only 60 iterations [163]. Mao and Zhao tested the results on sphericity error and uncertainty verification based on GPS using PSO, GA and LSM. It was revealed that PSO accuracy is greater than LSM and comparable with GA. The other advantage of PSO over GA is its faster convergence speed towards optimal solution [164].

Huang et al. proposed a hybrid optimization technique integrating modified cuckoo search algorithm and an adaptive fuzzy logic controller (MCSF) for improved evaluation of sphericity error by enhancing the exploration of basic cuckoo search. The fuzzy logic aid in controlling the step size of cuckoo search thus enhancing diversification in basic version. The proposed MCSF approach provides 0.5 times value of sphericity error in 60 iterations in comparison to other heuristic methods [165]. Similarly, improvement in cuckoo search is performed in the study of Jiang et al. by incorporating levy flights and selective random walk mechanism in evaluation of minimum zone sphericity error. Both improvements maintains diversity in cuckoo search technique and further improve the global search ability. The results on large datasets proved that improved cuckoo search found accurate results with improved convergence in comparison to classical CS [166]. The study performed by Chen et al. used DE algorithm for evaluation of sphericity error. The results proved that evaluation accuracy of sphericity error was improved in comparison to tradition LSM approach [167]. Balakrishna et al. proposed a support vector machined based method for evaluating deviation of spherical parts. Some unique characteristics of SVM is applied and it was explored that number of datasets have no influence on the accuracy of SVM. In addition, the result on different examples concludes that SVM based method produced accurate sphericity deviation in comparison to non-linear optimization approach [168]. Wang et al. proposed an improved IEA (IIEA) algorithm for determining minimum zone conicity error. The quasi-random sequences were utilized for initial population and further self-adaptive mutation is performed to maintain the balance of exploration and exploitation in proposed algorithm. The minimum zone conicity errors evaluated using IIEA method were accurate than CMM software by 20%-40% at a faster rate [169] (Table 5).

4 Discussions

The evaluation process of different form errors based on minimum zone method through different computational techniques are discussed with noteworthy studies. Moreover, there are still several points that need to be considered for understanding the complexity involved in evaluation of form errors. Several researchers have utilized computational techniques for the determination of accurate form errors in different components features that are beneficial in the adequate performance of the individual components. The past literature study shows that authors have developed and utilized traditional techniques as well as advanced optimization algorithms for solving the non-linear and complex problems of form errors in metrology. Beginning with planar surface features i.e., straightness and flatness, the major traditional computational techniques applied are computational geometric technique based on convex hull, region elimination search, successive quadratic programming, Vectorial method, adaptive and iterative neighborhood-based search strategy, Monte Carlo method, simplex search and data envelopment analysis. The good thing about these algorithms are they start with initial good solution and it improves over the course of iterations till the global optimum solution is reached. The results of all such techniques provides robust results and provides unique solution. However, these traditional techniques seems to be complex making implementation of the procedure difficult and time consuming in deriving the optimal solution. In addition, these algorithms provides sub-optimal solutions and the estimated value realized were only approximation owing to the improper setting of different critical parameters. Furthermore, these drawbacks are minimized by most of the advanced optimization algorithms considered for straightness and flatness evaluation. The advanced optimization algorithms considered were genetic algorithm (GA), particle swarm optimization (PSO), Differential evolution (DE), artificial bee colony (ABC), artificial fish swarm algorithm (AFSA), cuckoo search (CS) and improvement in these algorithms for obtaining optimal solutions. These algorithms are well known, and accepted optimization algorithm employed in different applications over the years by scholars and researchers of respective domains [170–176]. The GA and DE are known as evolutionary algorithms, while PSO, ABC, CS and AFSA are recognized as swarm intelligence-based algorithms. All aforementioned algorithms have good convergence rate, flexibility, simplicity, suitable in high dimensions problem and accuracy in obtaining global optimal solutions [177]. Due to these advantages, such algorithms are widely used in metrology form error evaluation and have given more accurate solutions that are by far superior to traditional computational geometry-based techniques as is evident from discussion of previous sections. Additionally, it overcomes the disadvantage of traditional algorithm to get stuck in local optimal solution by maintaining the trade-off between exploration and exploitation capability.

Since the importance of circular features are well known and discussed in literature, thus evaluation of circularity and cylindricity have become an integral and challenging task in metrology. The circular feature form errors were also determined by different traditional and advanced optimization algorithms. The traditional algorithms utilized were mostly similar to what used for planar feature form error evaluation. However, some new traditional techniques such as Hybrid coordinate point transformation and Chebyshev approximation, steepest descent search, Geometric Approximating Searching Algorithm (GASA) and Increment-simplex algorithm were utilized in computation of circularity and cylindricity. These algorithms are mostly an approximation and gradient based algorithms which found good results as compared to LSM, have simplicity, and requires less computation. However, the approximations and better results than LSM does not guarantee optimal solution for form errors, also their performance depends on some pre-set parameters. The gradient based algorithm found it difficult to obtain exact solution as the number of design variable increases, computational speed is low and may fall in local optimum. Now, to overcome these issues authors in past years utilized advanced optimization algorithms that successfully improved their results in case of circularity and cylindricity. Mostly, the improvement or hybridization of algorithms are proposed for maintaining the diversification and intensification in the new hybrid algorithm and thus improving the solution quality of circular feature form error. Some of the important hybrid or improved optimization algorithms employed for circularity and cylindricity evaluation are Hybrid simulated annealing (SA)- Hooke–Jeeves pattern search, Constriction factor PSO (CFPSO) algorithm, Improved harmony search (IHS) algorithm, Greedy sine cosine algorithm with differential evolution (HGSCADE), ABC algorithm with Tabu search, Hybrid PSO and Chaos search etc. These aforementioned hybridization and improvements were introduced owing to the drawbacks in working of algorithms based on single concepts such as slow convergence rate, local optimum stagnation, and

Table 5 Summary of Computational techniques on Sphericity and conicity error evaluation

Computational technique	Description	Recommendation	References
Computational geometric techniques	Traditional computational techniques	When compared with other methods in literature and have unique solution with faster computation time	[144]
Geometry optimization searching algorithm (GOSA)		The proposed method provides accurate, fast convergence and depends on pre-set factor δ	[145]
Minimum radial separation (MRS) method		Provides accurate sphericity error value with minimum computation time	[146]
Sequential quadratic programming (SQP) method		Greater accuracy and better computational time in comparison to LSM	[147]
Primal dual interior method		Better results and applied less computational effort for global optimal solution in comparison to SQP and heuristic method	[148]
Intersection chord method		Sphericity error determined were similar to the published methods, however significant reduction in computation time was achieved	[149, 150]
Asymptotic search method		Evaluated sphericity error accurately and efficiently for large number of extracted datasets	[151]
Branch and bound (B&B) algorithm		Computationally fast, effective and provides exact results	[152]
Method based on transformation matrices		Proposed method found quick and accurate results when compared with other traditional methods	[153]
Selective zone search technique		Provides accurate values and takes less computation time in comparison to non-linear optimization technique	[154]
Chebyshev approximation method		Proposed method showed 0.0032 as conicity error in comparison to 0.0035 by LSM	[155]
Branching and approximation algorithm		Provides same result as traditional technique while taking only 0.12 s	[156]
Geometrical optimization searching algorithm (GOSA)		Conicity error results were better than CMM results	[157]
Immune evolutionary algorithm (IEA)	Advanced optimization algorithms	IEA performed better than IGA in accuracy and takes only 2.2 s in comparison to 2.5 s and 8.6 s by IGA and EGA respectively	[158]
Particle swarm optimization (PSO) algorithm		When compared with IEA and GA on different datasets, the PSO algorithm provides improved conicity error value and enhanced convergence	[159]
Improved Genetic algorithm (GA)		Takes less computation time and provide accurate error value of 0.00967 which is significantly smaller than the minimum zone solution (0.0132)	[160]
GA, PSO, ACO based on worst case method		GA successfully evaluated and sphericity error, outperformed PSO, ACO and LSM	[161]
Heuristic search algorithm with feature points model (HAS-FPM)		Improved solution by nearly 60% in comparison to intersecting chord method, energy method and IEA	[162]
Whale optimization algorithm (WOA)		When compared with IEA, EGA and IGA, the WOA provides better results with comparable accuracy in only 60 iterations	[163]
PSO, GA and LSM		PSO accuracy is greater than LSM and comparable with GA. The other advantage of PSO over GA is its faster convergence speed towards optimal solution	[164]
Modified cuckoo search algorithm and an adaptive fuzzy logic controller (MCSF)		Provides 0.5 times value of sphericity error in 60 iterations in comparison to other heuristic methods	[165]
Improved cuckoo search		Improved cuckoo search found accurate results with improved convergence in comparison to classical CS	[166]
Differential evolution (DE) algorithm		Evaluation accuracy of sphericity error was improved in comparison to tradition LSM approach	[167]
Support vector machined method		SVM based method produced accurate sphericity deviation in comparison to non-linear optimization approach	[168]
Improved IEA (IIEA) algorithm		Accurate results than CMM software by 20–40% and at a faster rate	[169]

inadequate balance between exploration–exploitation. The simulated annealing algorithm has good exploration, however, lacks good exploitation capability thus combined with Hooke–Jeeves pattern search for improving it. The constriction factor was introduced in PSO algorithm for enhancing the convergence characteristics, harmony search improved by altering certain parameters for improving evaluation accuracy along with convergence. The classical sine cosine algorithm also suffers from insufficient exploitation and premature convergence which was provided by DE to provide adequate balance and improve the performance. Similarly, Tabu search and chaos enhances the global search ability in ABC and PSO algorithm respectively. These hybridization improves both convergence speed and solution quality that is evident from the results of circularity and cylindricity error improved values in comparison to their basic version and traditional computational techniques.

The sphericity and conicity errors have significant influence on the functional performance of rotational components, thus different traditional techniques were discussed in this literature. The new computational techniques considered includes Minimum radial separation (MRS) method, Branch and bound (B&B) algorithm, Sequential quadratic programming (SQP) method, Selective zone search etc. while others are already used in aforementioned form errors. The MRS method also a geometric based technique which provides mathematical approximation to obtain the solution. The branch and bound may provide suboptimal solution and spend much time in middle regions thus it never reaches to an optimal value [178]. The SQP method depends on the initial starting points thus effective in local search in design space and lacks exploration of design solution. The selective zone search method effectiveness depends on the zones considered for searching of solutions, In addition, for larger selection area, the computation time will also increase. The new advanced optimization algorithms apart from earlier considered for sphericity or conicity includes immune evolutionary algorithm (IEA) and its improvement, whale optimization algorithm (WOA) and support vector machine method. The IEA has good exploitation however, it also lacks search capability thus improvement is needed for enhancing the search and convergence performance. The WOA is one of the recent optimization algorithm that mimics the behaviour of humpback whales [179]. The WOA has effective balance between exploration and exploitation that is evident from the accurate results obtained for sphericity error. Moreover, the support vector machines are well-known machine learning techniques that focused on balancing the model accuracy and its prediction ability, utilized in effective prediction of form errors and provide results better than traditional techniques. In addition, researchers are advised to go through some recently proposed important advanced optimization algorithm that may be beneficial in enhancing the accuracy of form evaluation process [180–199]. The comparison of some of used computational techniques for various form error evaluation is reported in Table 6 based on certain important criteria.

5 Conclusions and Recommendations

The present review focused on providing detailed and comprehensive review on application of traditional methods as well as advanced optimization algorithms for evaluation of different form errors. In metrology, the accurate determination of these errors is challenging and imperative for enhancing the functional performance of features, components, and assemblies. Thus, evaluation of these form errors requires efficient methods and solution. In the same context, more than 150 literatures have been studied, examined, and analyzed using different computational techniques in evaluation of various form error. Based on above discussions and extensive literature review on different computational techniques i.e., traditional methods and advanced optimization algorithms, used for evaluation of metrology form errors, some suggestions and future directions are provided below:

1. Most of the traditional methods were based on geometry-based techniques that requires low computational effort, have moderate complexity, and have better accuracy than conventional LSM method.
2. Since the geometry-based techniques are higher, there is a good probability of encountering human error, in addition it may amplify further during its implementation as the iteration increases. Thus, it is suggested to apply or explore for more analytical models.
3. Also, a software based on these geometry-based techniques may also be developed having specific processing for individual techniques that will enhance the usage of such techniques in industries.
4. The classical version of advanced optimization algorithm such as GA, PSO, AFSA, ABC, DE and CS are mostly employed for realizing the solution of non-linear minimum zone fitness functions. It is worth noting that though all the algorithms are moderately accurate and requires similar computational effort in their implementation. However, as the number of datasets increased, the computational effort requirement and complexity may also increase.
5. It is advisable to linearize the non-linear fitness function of different form errors (such as using Taylor expansion) so that traditional methods may enhance their results.

Table 6 Comparison of various computational technique in form error evaluation

S. no	Computational Techniques	Local search	Global search	Complexity	Reliability	Computational effort
1	Geometric technique based on convex hull	Yes	No	Moderate	Moderate	Low
2	Successive quadratic programming	Yes	No	Low	Low	Low
3	Adaptive and iterative neighborhood-based search strategy	Yes	No	Moderate	Moderate	High
4	Geometric Approximating Searching Algorithm (GASA)	Yes	No	Moderate	High	Low
5	Data envelopment analysis	Yes	No	Moderate	Moderate	Low
6	Steepest descent search	Yes	No	Low	Low	Low
7	Increment-simplex algorithm	Yes	No	Low	Low	Low
8	Minimum radial separation (MRS) method	Yes	No	Moderate	Low	Moderate
9	Branch and bound (B&B) algorithm	Yes	No	Moderate	Moderate	Low
10	Genetic Algorithm (GA)	No	Yes	High	Moderate	Moderate
11	Particle swarm optimization (PSO)	No	Yes	Moderate	Moderate	Moderate
12	Differential evolution (DE)	No	Yes	High	Moderate	Moderate
13	Artificial bee colony (ABC)	No	Yes	Moderate	Moderate	Moderate
14	Artificial fish swarm algorithm (AFSA)	No	Yes	High	Moderate	Moderate
15	Cuckoo search (CS)	No	Yes	Low	Moderate	Low
16	Hybrid simulated annealing (SA)- Hooke–Jeeves pattern search	No	Yes	High	High	High
17	Improved harmony search (IHS) algorithm	No	Yes	High	High	High
18	Constriction factor PSO (CFPSO) algorithm	No	Yes	Moderate	Moderate	Moderate
19	Greedy sine cosine algorithm with differential evolution (HGSCADE)	No	Yes	High	High	High
20	ABC algorithm with Tabu search	No	Yes	High	Moderate	High
21	Immune evolutionary algorithm (IEA)	No	Yes	Moderate	Moderate	Moderate
22	Whale optimization algorithm (WOA)	No	Yes	High	High	Moderate
23	Support vector machine (SVM)	No	Yes	High	Moderate	Moderate

The complexity represents ease of implementation of algorithm. Computational effort represents convergence time to reach optimal solution

6. The hybrid advanced optimization algorithm proposed for different form error evaluation have higher complexity but at the same time provided more accurate results in comparison to classical algorithms and traditional methods owing to better balance between exploration and exploitation behaviour.
7. Thus, on seeing the effectiveness of finite hybrid optimization algorithms in efficiently obtaining the near exact values of form errors, it may be suggested to combine more traditional algorithms with classical single optimization algorithm for better results such as done with Hybrid simulated annealing (SA) and Hooke–Jeeves pattern search and AFSA-Powell method
8. There is an urgent need to use machine learning techniques and six sigma techniques for determination of form errors as very few have utilized such techniques for enhancing the overall accuracy.
9. Most of the studies on form error determination were based on extraction of data points using conventional CMM. However, with advancement of three-dimensional scanners and their accuracy in collecting point data from any surface, it is a challenge to use these data for evaluation of form error using some sampling techniques.
10. Most of the studies involve determination of straightness, flatness, circularity, cylindricity, sphericity using different techniques, however very few obtained the angular and conicity error. Thus, determination of these errors is significant for future studies as these features are now becoming common in most of industrial parts.

Declaration

Conflict of interest Authors declare that they have no conflict of interest.

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